Contents

1. Linear Expressions

1.1 Simplifying expressions

Algebra is the part of Mathematics in which we use letters to replace unknown numbers.

$3x + 5x - 4x + 2x$ is an **algebraic expression**, x is the **unknown**. The expression is **linear** because the highest power of x is 1. We also say it is an expression of **degree** 1.

$3x$, $5x$, $-4x$, $2x$ are all the same **power** and letter, we call them **like terms**.

We can **simplify** the expression by **collecting** like terms, $3x + 5x - 4x + 2x = 6x$.

EXAMPLE: Simplify $7x + 3 - 4x + 11 + 2x$

$7x + 3 - 4x + 11 + 2x = 7x - 4x + 2x + 3 + 11$
$= 5x + 14$

EXAMPLE: Simplify $5x + 4(5x + 3)$

(Remember: Expanding brackets from Module 1.)

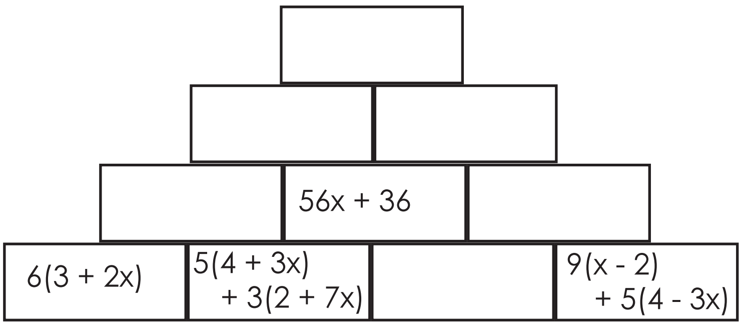
$5x + 4(5x + 3) = 5x + 20x + 12 =$
$25x + 12$

Practice

i. Simplify:

a. $3x + x + 4x + 2x$ **b.** $9y - 3y + 2y$ **c.** $9x + 2 - 7 + 3x$ **d.** $4x + 1 + 3x + 2 + x$
e. $5x - 2 - 3 - x$ **f.** $9x + 3y - 8x$ **g.** $2a - 6b - 8a$ **h.** $6x + 5y - 2x + 3y$
i. $-2z + 3y - 4y + 6z + x - 3y$ **j.** $2x + 4(x + 1)$ **k.** $3y + 3(y - 5)$
l. $6(2x - 3) + 2x$ **m.** $2(a + 4) + 3(a + 5)$ **n.** $8 - 3(2 - 5z)$

ii. The expression in each box is made by adding the two expressions below. In pairs fill in the missing expressions. Compare your answers with another group and correct any mistakes together.



Think

Some of the questions above involve adding, subtracting and multiplying using negative numbers. We studied this in module 1. Can you remember? Try to fill the gaps:

+ (+ a) = __ - (+ a) = - __
+ (___) = - a - (- a) = __

a x (- b) = - ___ (- a) x (___) = ab
(___) ÷ b = - ab (- a) ÷ (- b) = ___

The four rules above are called **general rules**. They are true for *all* numbers.

1.2 Factorising

Factorising is the opposite of expanding brackets. We can use factorising to simplify expressions. To factorise we look for common factors of the terms in the expression.

> **EXAMPLE:** Factorise 6x + 18
> 6x + 18 = 6x + 6 x 3 = 6(x + 3)

> **EXAMPLE:** Factorise 9ax + 36a
> 9ax + 36a = 9ax + 9a x 4 = 9a(x + 4)

Think

Look at the examples above. What do you notice about the numbers outside the brackets? (Hint: We studied this topic in Module 1). Complete the sentence: To factorise completely the ___________ ____________ __________ must be outside the brackets.

Practice

i. Find the Highest Common Factor (HCF) of these numbers

a. 6, 8 **b.** 12, 16 **c.** 5, 15 **d.** 24, 36 **e.** 2, 8, 10 **f.** x, 2x
g. 3x, 6x, 9x

ii. Factorise these expressions by finding the HCF of the terms

a. 5x - 20 **b.** 8x + 24 **c.** qx + 4q **d.** 16ax + 56a **e.** 9x + 12y **f.** 12x + 9y + 3

1.3 Algebraic fractions

In Module 1 we learnt how to simplify fractions by looking for common factors in the numerator and the denominator. We can simplify **algebraic fractions** in a similar way.

> **EXAMPLE:** Simplify $\frac{32zy}{56}$ in its simplest form
> $\frac{32zy}{56y} = \frac{4 \times 8 \times z \times y}{7 \times 8 \times y} = \frac{4z}{7}$

> **EXAMPLE:** Simplify $\frac{2a(a - b)}{a - b}$
> $\frac{2a(a - b)}{a - b} = \frac{2 \times a \times (a - b)}{a - b} = 2a$

Practice

Simplify

a. $\frac{2x}{8}$ **b.** $\frac{ab}{b}$ **c.** $\frac{3}{6a}$ **d.** $\frac{10x}{15xy}$ **e.** $\frac{21xyz}{49xz}$ **f.** $\frac{x - y}{x(x - y)}$

g. $\frac{4x}{8(x - y)}$ **h.** $\frac{12xy}{48y(x + y)}$ **i.** $\frac{8(x-y)}{12x(x - y)}$ **j.** $\frac{4 + a}{(4 + a)(6 - b)}$

Sometimes we must factorise before we can simplify.

> **EXAMPLE:** Simplify $\frac{12a - 4b}{3a - b}$
> $\frac{12a - 4b}{3a - b} = \frac{4(3a - b)}{3a - b} = 4$

Practice

a. $\frac{4a}{8a - 2b}$ **b.** $\frac{3a - 6b}{5a - 10b}$

c. $\frac{6x}{9x - 3xy}$ **d.** $\frac{24x - 12y}{6x - 2y}$

We also learnt in Module 1 that we add and subtract fractions by writing them as equivalent fractions with the same denominator, $\frac{2}{3} + \frac{1}{5} = \frac{2 \times 5}{3 \times 5} + \frac{1 \times 3}{5 \times 3} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$. We do the same with algebraic fractions.

> **EXAMPLE:** Simplify $\frac{3}{z} + \frac{4b}{y}$
> $\frac{3}{z} + \frac{2}{y} = \frac{3 \times y + 2 \times z}{zy} = \frac{3y + 2z}{zy}$

> **EXAMPLE:** Simplify $\frac{x + 3}{4} - \frac{x + 1}{3}$
> $\frac{x + 3}{4} - \frac{x + 1}{3} = \frac{3(x + 3) - 4(x + 1)}{12} =$
> $\frac{3x + 9 - 4x - 4}{12} = \frac{5 - x}{12}$

Practice

a. $\frac{1}{x} + \frac{1}{y}$ **b.** $\frac{1}{a} + \frac{1}{2b}$ **c.** $\frac{1}{2x} + \frac{1}{3x}$ **d.** $\frac{1}{8p} + \frac{1}{4q}$ **e.** $\frac{1}{2a} - \frac{1}{4b}$

f. $\frac{5}{8x} + \frac{2}{4y}$ **g.** $\frac{x + 2}{5} + \frac{x - 1}{4}$ **h.** $\frac{2x + 3}{4} - \frac{x - 2}{6}$ **i.** $\frac{x + 3}{7} - \frac{x + 2}{5}$

j. $\frac{3x + 1}{14} - \frac{2x + 3}{21}$ **k.** $\frac{1}{5}(4 - 3) - \frac{1}{10}(3 - x)$ **l.** $\frac{1}{9}(4 - x) - \frac{1}{6}(2 + 3x)$

m. $\frac{3(x - 1)}{4} + \frac{2(x + 1)}{3}$ **n.** $\frac{5(2x - 1)}{2} - \frac{4(x + 3)}{5}$

We also multiply and divide algebraic fractions in the same we way we mulitply and divide fractions.

Think

Write each step that is used to solve the division problem:

$\frac{a + 2b}{8} \div \frac{2a + b}{4}$ ____________________________________

$= \frac{a + 2b}{8} \times \frac{4}{2a + b}$ ____________________________________

$= \frac{4(a + 2b)}{8(2a + b)}$ ____________________________________

$= \frac{a + 2b}{2(2a + b)}$ ____________________________________

Practice

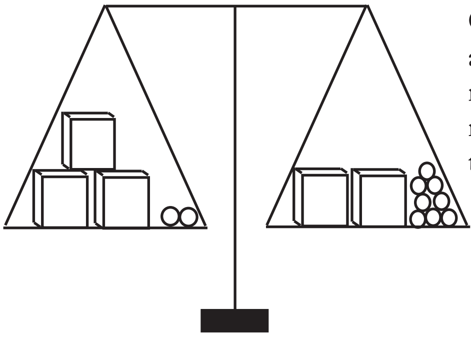
a. $\frac{2}{3} \times \frac{4}{5}$ **b.** $\frac{a}{b} \times \frac{c}{d}$ **c.** $\frac{x - y}{2} \times \frac{5}{x}$ **d.** $\frac{a}{b} \times c$ **e.** $\frac{p}{q} \div \frac{1}{r}$

f. $\frac{a - b}{4} \div \frac{a + b}{3}$ **g.** $\frac{x - 2}{3} \div (x + 3)$

2. Linear Equations

2.1 Solving linear equations

On this side there are 2 marbles and 3 boxes with an unknown number of marbles in each. The number of marbles in each box is the same.



On this side there are 8 marbles and 2 boxes with an unknown number of marbles in each. The number of marbles in each box is the same.

The scales can only balance if the two sides are equal.

Let x be the number of marbles in each bag. Algebraically we can write:

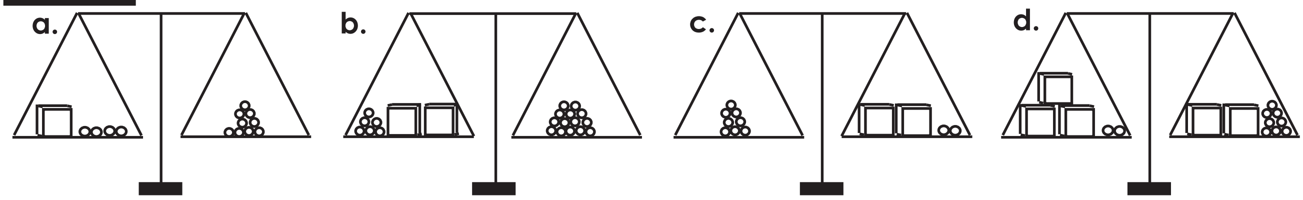
	$3x + 2 = 2x + 8$	The solution to the equation is $x = 6$.
Subtract $2x$ from both sides	$x + 2 = 8$	There are 6 marbles in each bag.
Subtract 2 from both sides	$x = 6$	

> When we are solving an **equation** we have to do the same to *both sides.*

Practice

In pairs write equations for x, the number of marbles in each box. Then find x.

Practice



EXAMPLE: Solve $11x - 6 = 8x + 9$

Subtract $8x$ from both sides	$3x - 6 = 9$
Add 6 to both sides	$3x = 15$
Divide by 3	$x = 5$

EXAMPLE: Solve $6(3x + 5) = 12 + 5x$

Expand the bracket	$8x + 30 = 12 + 6x$
Subtract $6x$ from both sides	$12x + 30 = 12$
Subtract 30 from both sides	$12x = -18$
Divide by 12	$x = -1.5$

i. Solve

a. $x + 3 = 7$ **b.** $9 + a = 15$
c. $5z - 9 = 16$ **d.** $19x - 16 = 22$
e. $9 = 6a - 27$ **f.** $8x + 11 = 3$

ii. Solve

a. $3x + 4 = 2x + 8$
b. $2x + 5 = 5x - 4$ **c.** $7x - 25 = 3x - 1$
d. $11x - 6 = 8x + 9$
e. $12 + 2x = 24 - 4x$ **f.** $9 - 3x = -5 + 4x$
g. $5 - 3x = 1 - x$ **h.** $4 - 3x = 1 - 4x$
i. $3 - 2x = 3 + x$

iii. Solve

a. $3x + 2x - 4x = 6$ **b.** $6x = x + 2 - 7 - 1$ **c.** $7 + 2x = 12x - 7x + 2$
d. $2 - 4x - x = x + 8$ **e.** $2x + 7 - 4x + 1 = 4$ **f.** $15x + 2x - 6x - 9x = 20$

Think

In this question the solution to the problem is given step-by-step. Next to each step write down what was done.

$$3(2x + 1) + 6 + 2x = 3x - 4 - 2$$
$$6x + 3 + 6 + 2x = 3x - 4 - 2 \quad \_\_\_\_\_\_\_\_\_\_\_\_$$
$$8x + 9 = 3x - 6 \quad \_\_\_\_\_\_\_\_\_\_\_\_$$
$$5x + 9 = -6 \quad \_\_\_\_\_\_\_\_\_\_\_\_$$
$$5x = -15 \quad \_\_\_\_\_\_\_\_\_\_\_\_$$
$$x = -5 \quad \_\_\_\_\_\_\_\_\_\_\_\_$$

Practice

Solve

a. $2(x + 4) = 3(2x + 1)$ **b.** $2(2x - 1) = 6$ **c.** $6(3x + 5) = 12$
d. $7(2x - 1) = 5(3x - 2)$ **e.** $4(3x + 2) = 14$ **f.** $x - 4(x + 3) = 3$
g. $5 - 3(x - 4) = 6$ **h.** $3p - 2 = 4 - 3(p + 2)$ **i.** $7(5 - x) = 3(x - 5)$
j. $5(x - 2) - 3(x + 1) = 0$ **k.** $3x - 4(1 - 3x) = 2x - (x + 1)$

2.2 Using linear equations

Here we will study how to use linear equations to solve word problems. There are 2 examples and some problems to solve.

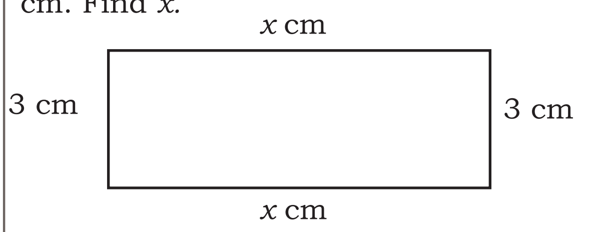
EXAMPLE: Aye Mon has x baht. Mi Chan has 6 baht less than Aye Mon. Together they have 30 baht. How much has Aye Mon?

The sum of Aye Mon and Mi Chan's money is 30 baht:

$$x + (x - 6) = 30 \text{ baht}$$
$$2x - 6 = 30 \text{ baht}$$
$$2x = 36 \text{ baht}$$
$$x = 18 \text{ baht.}$$

Aye Mon has 18 baht.

EXAMPLE: The sides of the rectangle are x cm and 3 cm. The perimeter is 24 cm. Find x.

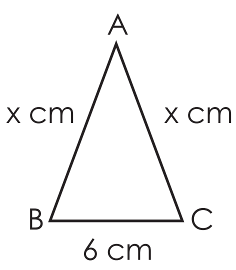


The perimeter is equal to the sum of the four sides:

$$x + x + 3 + 3 = 24$$
$$2x + 6 = 24$$
$$2x = 18$$
$$x = 9 \text{ cm}$$

Practice

Solve the following problems



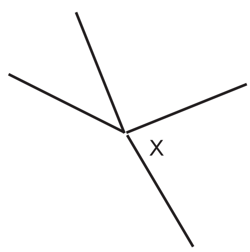
a. The perimeter of the triangle is 24 cm and AB = AC. Find AB.

b. Moe Thein has x dollars. George has 8 dollars more. Together they have 80 dollars. How much has Moe Thein?

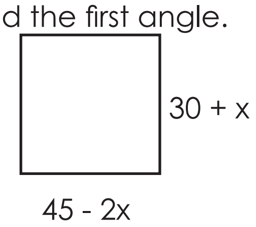
c. Gay Htoo has x baht. Hser Hser has 6 dollars more than Gay Htoo. Seik Nyan has 2 dollars more than Hser Hser. Together all three friends have 32 dollars. How much has Gay Htoo?

d. A rectangle has two sides of length 2x and 2 sides of length 3x + 2. The perimeter is 44 cm. Find x.

e. The first angle is x. The second angle is twice the first. The third angle is 30° and the fourth angle is 90°. Find the first angle.



f. The diagram shows a square. Find x.



2.3 Fractional equations

Some equations contain fractions. First we will learn how to solve **fractional equations** with the unknown as the numerator.

EXAMPLE: Solve the equation $\frac{2x}{5} = \frac{1}{3}$

Multiply both sides by 5: $2x = \frac{5}{3}$

Divide by 2: $x = \frac{5}{3} \div 2 = \frac{5}{3} \times \frac{1}{2} = \frac{5}{6}$

The solution is: $x = \frac{5}{6}$

EXAMPLE: Solve the equation $\frac{x}{5} + \frac{1}{2} = 1$

The lowest common multiple of 2, 5 is 10.
Multiply both sides by the LCM:
$10 \times \frac{x}{5} + 10 \times \frac{1}{2} = 10$

Cancel the common factors: $2x + 5 = 10$
Subtract 5 from both sides: $2x = 5$

Divide by 2: $x = \frac{5}{2}$

Practice

i. Solve:

a. $\frac{2x}{3} = 8$ **b.** $\frac{6x}{5} = 10$ **c.** $12 = \frac{3x}{2}$ **d.** $\frac{4x}{3} + \frac{1}{5}$ **e.** $\frac{5x}{7} - \frac{3}{4}$ **f.** $\frac{6x}{11} = \frac{5}{7}$

ii. Solve:

a. $\frac{x}{3} + \frac{1}{4} = 1$ **b.** $\frac{x}{5} - \frac{3}{4} = 2$ **c.** $\frac{x}{3} - \frac{2}{9} = 4$ **d.** $\frac{5x}{7} + \frac{x}{2} = 2$ **e.** $\frac{2x}{3} - \frac{1}{2} = 4$ **f.** $\frac{3x}{4} - \frac{x}{2} = 5$

In the following examples and questions we find the lowest common multiple of more than 2 numbers.

EXAMPLE: Solve the equation $\frac{x}{5} + \frac{2}{3} = \frac{14}{15}$

The LCM of 5, 3, 15 is 15.
Multiply both sides by 15: $15 \times \frac{x}{5} + 15 \times \frac{2}{3} = 14$

Cancel the common factors: $3x + 10 = 14$
Subtract 10 from both sides: $3x = 4$
Divide by 3: $x = \frac{3}{4}$

EXAMPLE: Solve the equation $\frac{x}{5} + \frac{x+1}{4} = \frac{8}{5}$

The LCM of 5, 4, 5 is 20.
Multiply both sides by 20: $20 \times \frac{x}{5} + 20 \times \frac{x+1}{4} = 32$

Cancel the common factors: $4x + 5(x + 1) = 32$
Expand the brackets and simplify: $9x + 5 = 32$
Subtract 5 from both sides: $9x = 27$
Divide by 9: $x = 3$

Practice

i. Solve:

a. $\frac{x}{3} + \frac{1}{4} = \frac{1}{2}$ **b.** $\frac{x}{4} - \frac{1}{2} = \frac{9}{4}$

c. $\frac{x}{2} - \frac{3}{7} = \frac{1}{2}$ **d.** $\frac{5x}{6} + \frac{x}{8} = \frac{3}{4}$

e. $\frac{5x}{12} - \frac{1}{3} = \frac{x}{9}$ **f.** $\frac{2x}{5} - \frac{x}{15} = \frac{5}{9}$

g. $\frac{3x}{4} + \frac{1}{3} = \frac{x}{2} + \frac{5}{8}$ **h.** $\frac{2x}{9} - \frac{3}{4} = \frac{7}{18} - \frac{5x}{12}$

i. $\frac{3}{5} - \frac{x}{9} = \frac{2}{15} - \frac{2x}{45}$

ii. Solve

a. $\frac{x}{4} - \frac{x+3}{3} = \frac{1}{2}$ **b.** $\frac{2x}{5} - \frac{x-3}{8} = \frac{1}{10}$

c. $\frac{x+3}{5} - \frac{x-2}{4} = \frac{1}{10}$

d. $\frac{x+2}{4} + \frac{x-3}{2} = \frac{1}{2}$

e. $\frac{2}{3} - \frac{x+1}{9} = \frac{5}{6}$

iii. Solve:

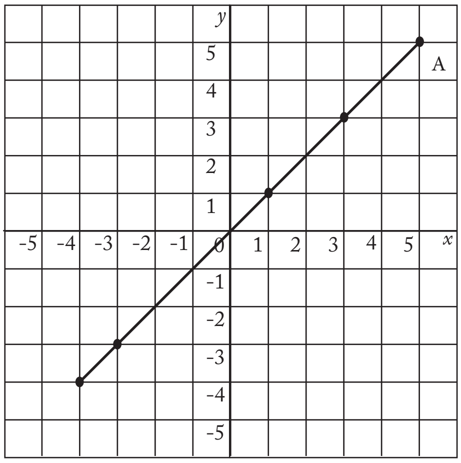
a. $\frac{1}{2} + \frac{4}{x} = 1$ **b.** $\frac{2}{3} - \frac{1}{x} = \frac{13}{15}$

c. $\frac{3}{8} + \frac{2}{x} = \frac{1}{6}$ **d.** $\frac{3}{2x} + \frac{1}{4} = \frac{1}{3}$ **e.** $1 - \frac{1}{2} = \frac{3}{2x}$ **f.** $\frac{3}{2x} + \frac{2}{5} = 5$

3. Graphing Linear Equations

3.1 Introduction

Look at point A on the **graph**. It has the **coordinates** (5,5). The **x-coordinate** is 5 and the **y-coordinate** is 5. We can draw a table of the coordinates of each point.



x	y	(x,y)
-4	-4	(-4,-4)
-3	-3	(-3,-3)
1	1	(1,1)
3	3	(3,3)
5	5	(5,5)

We can see that for each point the x-coordinate is the same as the y-coordinate, i.e.

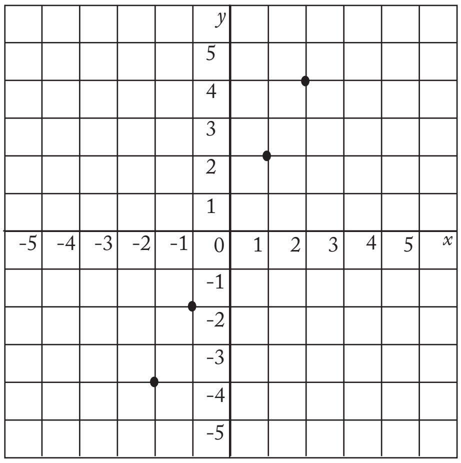
x-coordinate = y-coordinate

$y = x$

$y = x$ is the equation of the line.

Think

i. Complete the table to give the coordinates of each point on the graph.



x	y	(x,y)
-2		
	2	

ii. Draw a straight line through the points.

iii. What is the relationship between the x-coordinate and the y-coordinate of each point?

iv. What is the equation of the line

Practice

i. Complete the table for each given equation.

a. $y = 2$

x	y	(x,y)
-5		
-2		
0		
1		
3		

b. $y = -x$

x	y	(x,y)
-5		
-2		
0		
1		
3		

c. $y = 3x$

x	y	(x,y)
-5		
-2		
0		
1		
3		

d. $y = x/2$

x	y	(x,y)
-5		
-2		
0		
1		
3		

e. $y = x + 1$

x	y	(x,y)
-5		
-2		
0		
1		
3		

f. $y = 3x + 1$

x	y	(x,y)
-5		
-2		
0		
1		
3		

g. y = 7x + 6

x	y	(x,y)
-5		
-2		
0		
1		
3		

h. y = 2 - x

x	y	(x,y)
-5		
-2		
0		
1		
3		

i. y = 5 - 3x

x	y	(x,y)
-5		
-2		
0		
1		
3		

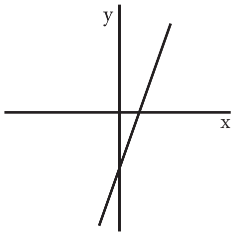
ii. On graph paper plot the points for each set of coordinates a. - i. and draw the straight line.

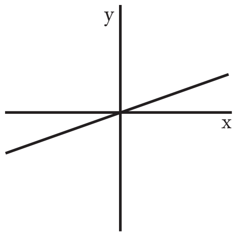
iii. Use the graphs to find the value of y when x = 0 for each equation.

3.2 The gradient of a line

You will notice that the graphs in the previous question have different slopes or **gradients**.

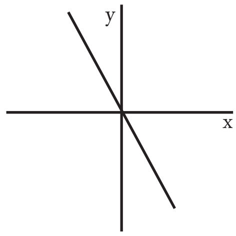
Some lines slope steeply to the right.
They have *large positive* gradients.





Some lines have a shallow slope to the right.
They have a *small positive* gradient.

Some lines slope to the left.
They have a *negative* gradient.



To find the gradient of a line we choose two points and calculate: $\frac{\text{the difference in the y-coordinates}}{\text{the difference in the x-coordinates}}$

EXAMPLE: Find the gradient of the line $y = -2x + 3$

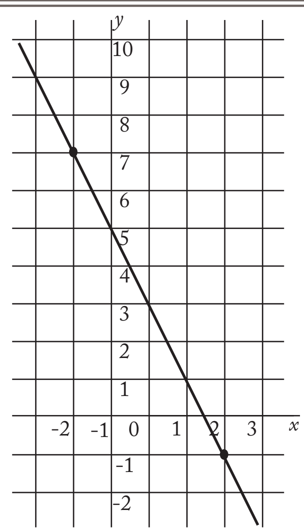
Write a table of values for coordinates of points on the line

x	y	(x,y)
-2	7	(-2,7)
0	3	(0,3)
1	1	(1,1)
2	-1	(2,-1)

If we choose points (-2,7) and (2,-1) then we have

Gradient = $\frac{\text{the difference in the y-coordinates}}{\text{the difference in the x-coordinates}}$

$= \frac{-1 - 7}{2 - (-2)} = \frac{-8}{4} = -2$



Practice

i. Use the method in the example to find the gradient of each line:

a. y = 2x + 3 **b.** y = 2x - 1 **c.** y = x + 4 **d.** y = 3 - 2x

ii. Look at your answers to i. What do you notice about the value of the gradient for each line?

iii. Give the gradient of the line y = 4x + 1 without any calculation.

3.4 Using graphs to solve problems

We can use graphs to solve real-life problems.

EXAMPLE: A woman walks at a speed of 4 kilometres an hour.

a. Make a table of values of time travelled, x against distance travelled, y

b. Draw the graph of the table of values

c. Use your graph to find how far the woman walks in 3 1/2 hours

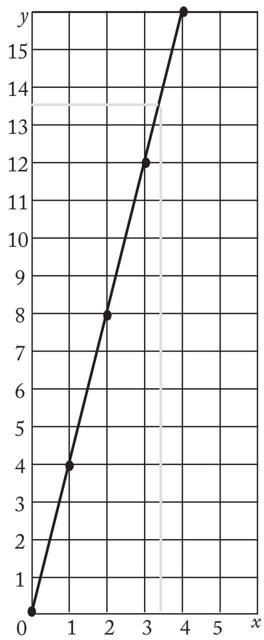
ANSWERS:

a. The completed table is

Time travelled in hours, x	0	1	2	3	4
Distance travelled in km, y	0	4	8	12	16

b. If we plot the points in the table we have the graph on the right (note that time and distance are positive so we only need positive axes).

c. If we draw a vertical line from $x = 3\ 1/2$ to the graph and then draw a horizontal line to the y-axis, we see that the woman walks 13 1/2 km in 3 1/2 hours.



Practice

Solve the following problems

i. A boat travels on the Salween river at a speed of 3 kilometres an hour.

a. Complete this table of values

Time travelled in hours, x	0	1	2	3	4
Distance travelled in km, y	0	3			

b. Draw the graph for the table of values

c. Use the graph to find how far the boat travelled in 2 1/2 hours.

ii. In Myanmar miles are used to measure distance. In Thailand kilometres are used. To change between miles and kilometres we can use the approximate relationship 5 miles = 8 kilometres.

a. Complete this table of values

Time travelled in hours, x	0	5	10	15	20	25
Distance travelled in km, y	0	8				

b. Draw the graph for the table of values

c. How many miles is approximately the same as 14 kilometres?

iii. To cook rice Thee Thee uses 1.5 cups of water for every cup of rice.

a. Complete this table of values

Number of cups of rice, x	1	2	3	4	5	6
Number of cups of water, y	1.5					

b. Draw the graph for the table of values

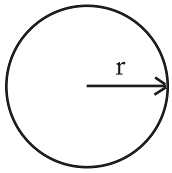
c. How many cups of water does Thee Thee use to make 5.5 cups of rice?

4. Formulae

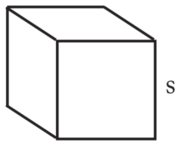
4.1 Introduction

A **formula** is a rule that gives a relationship between unknowns (the plural for formula is **formulae**). Here are some examples of formulae that you may have seen before.

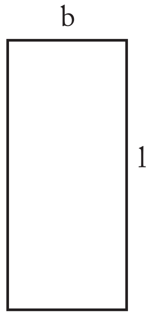
Distance = speed x time



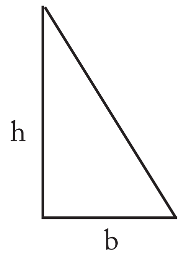
Circumference = 2Pr
Area = Pr^2



Volume = s^3



Area = l x b



Area = $\frac{1}{2}$b

Look at the difference between a formula and an equation:

$C = \pi \times d$ is a formula

Substituting any value for d will give a value for C.
Substituting any value for C will give a value for d.

C = π x 2.5 is an equation

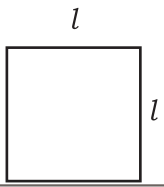
The equation is only true for one value of C:
C = 7.85 (to 2 d.p.)

4.2 Constructing formulae

We can use given information to construct formulae.

EXAMPLE: The perimeter of the square is P. Write a formula for P.

$P = l + l + l + l$
$P = 4l$



EXAMPLE: Sam and Oo are playing caneball. If one player wins a game they get 3 points. If the game is a draw*, both players get 1 point. Write a formula for Sam's total score, T_s, if he has a wins and b draws.

We multiply the number of wins by 3 points and add the number of draws to get:

$$T_s = 3a + b$$

*A draw means all scores were equal at the end of the game.

EXAMPLE: Aung Aung goes to the market and buys, x kg of potatoes, y g of coffee and $(x + a)$ kg of rice. Write a formula in grams for the total weight, T.

If we add all three quantities we get:
$T = 1000x + y + 1000(x + a)$
$= 1000x + y + 1000x + 1000a$
$= 2000x + 1000a + y$
$= 1000(2x + a) + y$

Note: We multiply x and $(x + a)$ by 1000, because the question asks for the answer in grams.

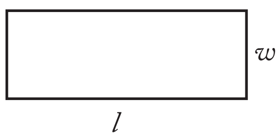
Think

Oo has three wins less than Sam and 4 draws more. Write a formula for Oo's total score, $T_{0.}$

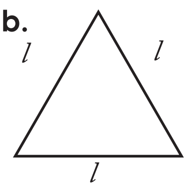
Practice

i. Write a formula for the perimeter, P of each shape. Simplify your answers.

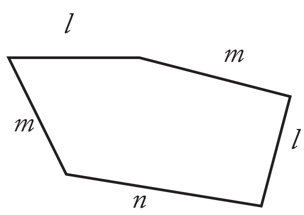
a.



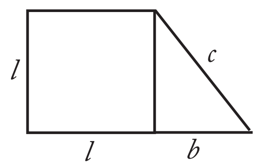
b.



c.



d.



ii. Write a formula for the area, A, of shape **d.**

iii. To visit his friend at the university Min Thaw walks x m to the road and then travels y km by bus. The university is (x - a) m from the bus stop. How far does he travel in kilometres?

iv. Do this exercise in pairs.

a. In the English football league teams score 3 points for a win and 1 point for a draw. At the end of 2007 Manchester United had x wins, y draws and z losses. Write a formula for their total number of points, T_m.

b. Now copy the table into your books and complete it using the information below:

Team	Win	Draw	Lose
Manchester United	x	y	z
Chelsea		y + 6	
Liverpool			
Arsenal			z + 3

Chelsea won 4 games less, drew 6 games more and lost 2 less games than Manchester United. Liverpool won 8 games less, drew 3 games more and lost 5 more games than Manchester United. Arsenal won 9 games less, drew 6 games more and lost 3 more games than Manchester United.

c. Write formulae for Chelsea (T_c), Liverpool (T_L) and Arsenal's (T_A)total number of points.

4.3 Evaluating formulae

We can **evaluate** formulae by substituting values. The values can be positive, negative or fractional.

EXAMPLE: $x = L(1 + at)$.
Find x when $L = 100$, $a = 0.19$ and $t = 80$.
Substituting the values gives:
$x = 100(1 + 0.19 \times 80) = 100(1 + 15.2)$
$= 100(16.2) = 1620$

EXAMPLE: The formula for the perimeter of a rectangle is $P = 2(l + w)$.
Find P when $l = 8\frac{1}{2}$ and $w = 6\frac{1}{4}$

$P = 2\left(8\frac{1}{2} + 6\frac{1}{4}\right) = 2 \times 14\frac{3}{4} = 29\frac{1}{2}$

EXAMPLE: $L = P(2 - a)$.
Find a when $L = 10$ and $P = 40$.
First we make a the subject of the equation:
Divide by P: $\frac{L}{P} = 2 - a$

Add a to both sides and subtract $\frac{L}{P}$ from both sides:
$a = 2 - \frac{L}{P}$

Now we substitute $L = 10$ and $P = 40$:
$a = 2 - \frac{10}{40} = 2 - \frac{1}{4} = 1\frac{3}{4}$

Practice

i. Look back at question iv. on the above. Use the values x = 28, y = 5 and z = 5 and your formulae T_M, T_C, T_L and T_A to evaluate the points gained by each team.

ii. For each question find the value of the unknown by rearranging each formula and substituting the given values:

a. If N = 2(p + q), find q when N = 24 and p = 5

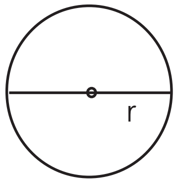
b. If $s = \frac{1}{3}(a - b)$, find b when s = 15 and a = 24

c. If $d = \frac{1}{2}(a + b + c)$ find a when d = 16, b = 4, c = -3

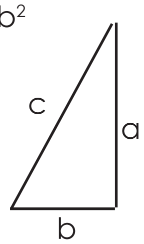
d. If H = P(Q - R), find Q when H=12, P=4 and R= -6

iii. Evaluate each formula to find the specified unknown. Give your answers to 2 decimal places.

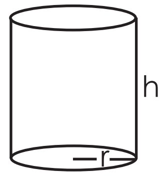
a. $C = 2\pi r$
Find C when
$r = 3.9$ and
$\pi = 3.142$



b. $c = \sqrt{a^2 + b^2}$
Find c when
$a = 3$ and
$b = 4$



c. $C = 2\pi rh$
Find h when
C = 801,
r = 6 and
$\pi = 3$ to the nearest whole number.



iv. Temperature can be measured in two units, degrees Celsius (oC) and Fahrenheit (F). A temperature given in Celsius can be changed to Fahrenheit using the formula:

$$F = \frac{9C}{5} + 32$$

The table below gives the temperature for 6 cities. Use the formula to complete the information. Give your answers to the nearest whole number.

	Beijing	Bangkok	Tokyo	New York	London	Moscow
°C	21			-7	3	
°F		95	74			63

5. Simultaneous Equations

5.1 Introduction

In chapter 2 we learnt how to solve equations with one unknown. Some equations have two or more unknowns. Consider the problem: 'Find two numbers with a sum of 8'. Algebraically we can write:

$$x + y = 8$$

Where x and y are the unknown numbers we are looking for. There are different solutions to this equation, such as x = 6, y = 2 or x = 5.5, y = 2.5. In fact, there are an **infinite** number of solutions.

Now consider a different problem 'Find two numbers with a difference of 8'. Here we can write:

$$x - y = 8$$

Again, there are an infinite number of solutions to this equation.

If we combine the two sentences, then we have 'Find two numbers with a sum of 8 and a difference of 8, then we have a pair of **simultaneous equations**:

$$x + y = 8$$

$$x - y = 8$$

This pair of equations have one set of solutions for x and y. We will learn how to find this solution in the following sections.

5.2 Solving by elimination

The numbers in front of unknowns in an equation are called **coefficients.** If the coefficients of 1 unknown in a pair of simultaneous equations are the same then we can add or subtract the equations. This **eliminates** one unknown and allows us to find the solution.

EXAMPLE: Solve the equations

$x + y = 5$ [1]

$3x + y = 7$ [2]

Equation [2] - Equation [1] gives

$2x = 2$

Divide by 2 $x = 1$

We substitute this value of x into [1]

$1 + y = 5$

$y = 4$

We check the answer by substituting in [2]

$3 + 4 = 7$

This is correct so the solution is $x = 1$, $y = 4$

EXAMPLE: Solve the equations

$x - 2y = 1$ [1]

$3x + 2y = 19$ [2]

Equation [1] + Equation [2] gives

$4x = 20$

Divide by 4 $x = 5$

We substitute this value of x into [2]

$15 + 2y = 19$

$2y = 4$

$y = 2$

We check the answer by substituting in [1]

$5 - 4 = 1$

This is correct so the solution is $x = 5$, $y = 2$

Practice

Solve the following pairs of equations:

i. a. x + y = 5 and 4x + y = 14
b. 2a + b = 11 and 4a + b = 17
c. 2x + 3y = 23 and x + 3y = 22
d. 5x + 2y = 14 and 7x + 2y = 22
e. x + 2y = 12 and x + y = 7
f. 4p + 3q = -5 and 7p + 3q = -11

ii. a. x - y = 2 and x - 2y = 1
b. p + 2q = 11 and 3p - 2q = 1
c. 4x + y = 37 and 2x - y = 17
d. 5p + 3q = 5 and 4p - 3q = 4
e. 3x - 4y = -24 and 5x + 4y = 24
f. 5x - 2y = 4 and 3x + 2y = 12

iii. a. $2x - y = 4$ and $x - y = 1$

b. $x - y = 3$ and $3x - y = 9$

c. $3p + 5q = 17$ and $4p + 5q = 16$

d. $2x - 3y = 14$ and $4x - y = 18$

e. $5x - 2y = -19$ and $x - 2y = -7$

f. $3x - y = 10$ and $x + y = -2$

If the coefficients of the unknowns are not the same, then we use a slightly different method

Consider the equations:

$$2x + 3y = 4 \quad [1]$$
$$4x + y = -2 \quad [2]$$

If we add these equations then we have: $6x + 4y = 2$. We have one equation with two unknowns and we cannot find the solution. However, if we multiply Equation [2] by 3 then we have:

$$2x + 3y = 4 \quad [1]$$
$$12x + 3y = -6 \quad [3]$$

Equation [3] - Equation [1] eliminates y to give

$$10x = -10$$

Dividing by 10 gives:

$$x = -1$$

We substitute this $x = -1$ into [1] to get $-2 + 3y = 4$. Which gives us $y = 2$. The solution is $x = -1$ and $y = 2$. We check the answer by substituting into [2]: $-4 + 2 = -2$. This is true, so our solution is correct.

EXAMPLE 1: Solve the equations

$$3x - 2y = 1 \quad [1]$$
$$4x + y = 5 \quad [2]$$

Multiply [2] by 2

$$8x + 2y = 10 \quad [3]$$
$$3x - 2y = 1 \quad [1]$$

Equation [3] + Equation [1] gives

$$11x = 11$$

Divide by 11 $x = 1$

We substitute this value of x into [2]

$$4 + y = 5$$
$$y = 1$$

We check the answer by substituting in [1]

$$3 - 2 = 1$$

This is correct so the solution is $x = 1$, $y = 1$

EXAMPLE 2: Solve the equations

$$3x + 5y = 6 \quad [1]$$
$$2x + 3y = 5 \quad [2]$$

Multiply [1] by 3

$$9x + 15y = 18 \quad [3]$$

Multiply [2] by 5

$$10x + 15y = 25 \quad [4]$$

Equation [4] - Equation [1] gives

$$x = 7$$

We substitute this value of x into [2]

$$14 + 3y = 5$$
$$3y = -9$$

Divide by 3 $y = -3$

We check the answer by substituting in [1]

$$21 + (-15) = 6$$

This is correct so the solution is $x = 7$, $y = -3$

Practice

i. Use Example 1 to solve the following pairs of equations:

a. $2x + y = 7$ and $3x + 2y = 11$

b. $5x - 4y = -3$ and $3x + y = 5$

c. $5x + 3y = 21$ and $2x + y = 3$

d. $6x - 4y = -4$ and $5x + 2y = 2$

e. $5x + 3y = 11$ and $4x + 6y = 16$

f. $2a - 3b = 1$ and $5a + 9b = 19$

g. $9x + 5y = 15$ and $3x - 2y = -6$

h. $4x + 3y = 1$ and $16x - 5y = 21$

ii. Use Example 2 to solve the following pairs of equations

a. $2x + 3y = 12$ and $5x + 4y = 23$

b. $2x - 5y = 1$ and $5x + 3y = 18$

c. $5x + 4y = 21$ and $3x + 6y = 27$

d. $9x + 8y = 17$ and $2x - 6y = -4$

e. $2x - 6y = -6$ and $5x + 4y = -15$

f. $6a + 5b = 8$ and $3a + 4b = 1$

g. $6x - 5y = 17$ and $5x + 4y = 6$

h. $17x - 2y = 47$ and $5x - 3y = 9$

5.3 Solving by substitution

We can also solve pairs of simultaneous equations by **substitution.**

EXAMPLE: Solve the equations:

$2x - y = -10$ [1]
$3x + 2y = -1$ [2]

Write [1] in terms of y:
$y = 2x + 10$
Substitute in [2]:
$3x + 2(2x + 10) = 25$
Simplify: $7x + 20 = -1$
Find x: $7x = -21$
$x = -3$
Substitute this value of x into [1]: $-6 - y = -10$
$y = 4$
Check by substituting in [2]: $-9 + 8 = -1$

This is true so the solution is $x = -3$, $y = 4$

Think

Use the example to complete the steps and solve the pair of equations:
x - y = 8 [1]
2x + y = 7 [2]
Write [2] in terms of y:
____________________[3]
Substitute [3] into [1]:
____
Simplify:
____
Find x:
____
Substitute in [1] to find y:
____
Check the solution by substituting into [2]: ____________________

Practice

Use substitution to solve the following pairs of equations

a. x = 4 and x + y = -3 **b.** m = -6n and -2m - 4n = -40 **c.** x + y = -3 and x - y = -3
d. x - y = -7 and 3x - 2y = -16 **e.** s + t = 5 and 2s + 3t = 6
f. 3a - 5b = 10 and 6a - b = 2 **g.** x + y = 100 and 0.5x + 0.6y = 55
h. 3x + 6y = 12 and -2x + 5y = 1 **i.** 1.5x - 0.5y = 4 and 1.5x - 2.5y = 2

5.4 Using simultaneous equations

Here we will study how to use simultaneous equations to solve word problems.

EXAMPLE: A shop sells noodles and fried rice. 3 bowls of noodles and 4 plates of rice cost 1420 kyat. 2 bowls of noodles and 3 plates of rice cost 1010 kyat. Find the cost of a bowl of noodles and a plate of rice.

Let the cost of a bowl of noodles be x and the cost of a plate of rice be y.
3 bowls of noodles and 4 plates of rice = $3x + 4y = 1420$ kyat [1]
2 bowls of noodles and 3 plates of rice = $2x + 3y = 1010$ kyat [2]

Equation [1] x 3 and Equation [2] x 4 gives
$9x + 12y = 4260$ baht [3]
$8x + 12y = 4040$ baht [4]
[3] - [4] gives $x = 220$ baht
Substituting in [1] gives $660 + 4y = 1420$, $4y = 760$, $y = 190$ baht
Check in equation [2]: 440 + 570 = 1010. This is true to the solution is correct.
One bowl of noodles costs 220 baht and a plate of fried rice costs 190 baht.

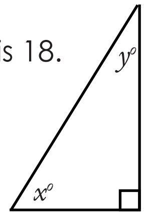
Practice

Solve the following problems by forming a pair of simultaneous equations

a. The sum of two numbers is 20 and their difference is 4. Find the two numbers.
b. The sum of two numbers is 16 and their difference is 6. Find the two numbers.
c. Three times a number added to a second number is 33. The first number added to three times the second number is 19. Find the two numbers.
d. The total weight of 3 elephants and 2 giraffes is 2.5 tonnes. The total weight of 5 elephants and 4 giraffes is 5.2 tonnes. Find the weight of one elephant.
e. 1000 tickets were sold for an Ironcross concert in Yangon. An adult ticket cost 850

kyat and a child ticket cost 450 kyat. The total money from ticket sales was 730,000 kyat. Let x be the number of adult tickets sold and y be the number of child tickets sold. Find the number of adult tickets sold.

f. x is greater than y. The difference between x and y is 18. Find x and y.



g. The perimeter of the rectangle is 31cm. The difference between the lengths AB and BC is 3.5cm. Find the lengths AB and BC.

h. Work through this problem in pairs.

Sequence 1	5	7	9	11	13	15	17	19	21	23
Sequence 2	-23	-20	-17	-14	-11	-8	-5	-2	1	4
Sequence 3	56	49	42	35	28	21	14	7	0	-7

The table shows 3 three sequences of numbers that increase or decrease by the same amount.

i. Choose any six consecutive numbers from a sequence (e.g. 49, 42, 35, 28, 21, 14) and place them in order into the pair of equations and solve the equations.

____ x + ____ y = ____

____ x + ____ y = ____

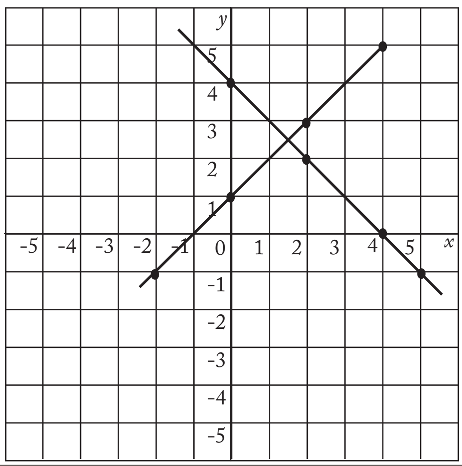
ii. Choose six different numbers from a sequence and solve again. What do you notice?

5.5 Solving by drawing graphs

We saw in Chapter 3 that we can draw graphs of linear equations of the form $y = mx + c$, where m and c are numbers. Equations of this form give straight line graphs. If we have two equations we have will have two straight lines. The point where they cross gives us the solution to the pair of equations.

EXAMPLE: Draw a graph to solve the equations:

$x + y = 4$ $\quad\quad$ $y = 1 + x$



If we look at the graph we can see that the lines cross at the point (1.5, 2.5). This gives the solution $x = 1.5$, $y = 2.5$

Practice

Solve the following pairs of equations by drawing graphs.

a. x + y = 6 and y = 3 + x

b. y = 4 + x and y = 1 + 3x

c. 2x + y = 3 and x + y = 2 1/2

d. y = 5 - x and y = 2 + x

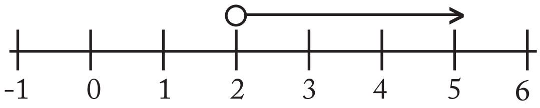
6. Linear Inequalities

6.1 Introduction

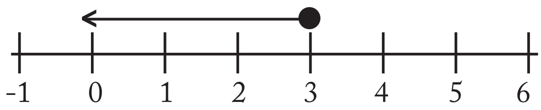
An expression in which the left hand side and the right hand side are *not* equal is called an **inequality**.
These four expressions are all inequalities.

$x > 2$ means x is greater than 2
$x < 2$ means x is less than 2
$x \geq 2$ means x is greater than or equal to 2
$x \leq 2$ means x is less than or equal to 2

Inequalities can be shown on a number line



This number line shows $x > 2$.
The circle is open because 2 is not included.

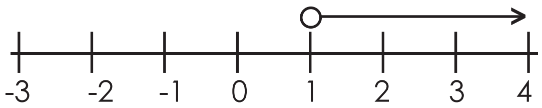


This number line shows $x \leq 3$.
The circle is closed because 3 is included.

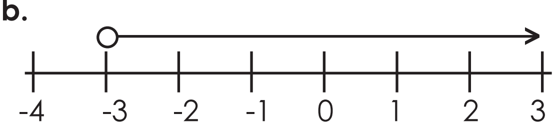
Practice

i. Write the inequalities shown on the number lines:

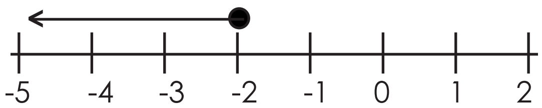
a.



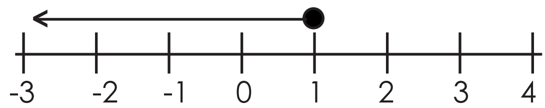
b.



c.



d.



ii. Draw number lines to show the inequalities:

a. $x < 4$ **b.** $x > -2$ **c.** $x \leq -1$ **d.** $x < 4$
e. $x \leq 0$ **f.** $x > 0.5$

Think

$3 > -6$ is true. It is a true inequality. Complete each of the operations and say whether the inequality remains true. The first is completed as an example.

a. Add 4 to both sides	$3 + 4 > -6 + 4$	
	$7 > 2$	<u>True</u>
b. Subtract two from both sides	3 - ___ > -6 - ___	
	___ > ___	____
c. Multiply both sides by 2		
		____
d. Divide both sides by 3		
		____
e. Multiply both sides by -5		
		____
f. Divide both sides by -3		
		____

Now, complete the statements to give the rules for solving inequalities.

To solve an inequality you **can**:

__________ the same number to both sides.

__________ the same number from both sides.

Multiply both sides by the same __________ number.

Divide both sides by the same __________ number.

To solve an inequality you **cannot**:

Multiply both sides by the same __________ number.

Divide both sides by the same __________ number.

6.2 Solving inequalities

To solve an inequality we rearrange it so that the x term is on one side. This gives a range of values for x. Always remember the rules from the previous exercise.

EXAMPLE: Solve $3x + 4 > 22$

Subtract 4 from both sides	$3x > 18$
Divide by 3	$x > 6$

EXAMPLE: Solve $6 - 5x \geq 3x + 2$

Subtract 2 from both sides	$4 - 5x \geq 3x$
Add $5x$ to both sides	$4 \geq 8x$
Divide by 4	$1 \geq 2x$

Practice

i. Solve the following inequalities

a. $x - 4 < 8$ **b.** $x + 2 < 4$ **c.** $x - 5 < -2$ **d.** $x + 7 < 0$

e. $4 - x > 6$ **f.** $2 > 5 + x$ **g.** $5 - x < -7$

ii. Solve the following inequalities

a. $3x - 2 < 7$ **b.** $1 + x > 3$ **c.** $4x - 5 < 4$ **d.** $6x + 2 > 11$

e. $5 + 2x < 7$ **f.** $3 < 5 - 2x$ **g.** $4 - 3x < 10$ **h.** $10 < 3 - 7x$

i. $x - 1 > 2 - 2x$ **j.** $3x + 2 < 5x + 2$ **k.** $2x - 5 > 3x - 2$

If an inequality has brackets, we expand them first and then solve as above.

EXAMPLE: Solve $2(x + 2) + 3(2x - 3) > 2$

Expand the brackets	$2x + 4 + 6x - 9 > 2$
Simplify the left hand side	$8x - 5 > 2$
Add 5 to both sides	$8x > 7$
Divide both sides by 8	$x > 7/8$

Practice

Solve the following inequalities

a. $3(x - 7) > 14$ **b.** $5(x + 4) > 40$ **c.** $4(x + 1) \leq 2x - 3$ **d.** $7(x - 2) - (5x - 9) \geq 0$

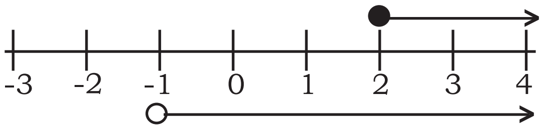
e. $3x + 4(2x + 3) < 1$ **f.** $6(3x + 1) - 2(5x - 4) < 5(x + 2)$ **g.** $3(4x + 5) + (8 - 10x) \leq 4(x + 6)$

6.3 Range of values

If we have two inequalities we can find a **range of values** for which both inequalities are true.

EXAMPLE: Find the range of values which satisfy the inequalities $x \geq 2$ and $x > -1$.

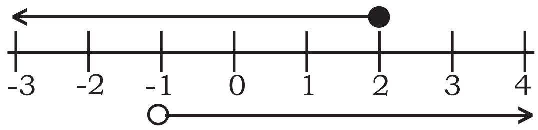
Draw both inequalities on a number line.



We can see from the number line that x > 2 and x > -1 are both satisfied in the range: $x \geq 2$

EXAMPLE: Find the range of values which satisfy the inequalities $x \leq 2$ and $x > -1$.

Draw both inequalities on a number line.



We can see from the number line that $x \leq 2$ and $x > -1$ are both satisfied in the range: $-1 < x \leq 2$

Practice

Try to find the range of values for which the two inequalities are true:

a. x > 2 and x > 3 **b.** x ≥ 2 and x ≤ 3
c. x > 4 and x < -2 **d.** x < 4 and x > -2
e. x < 0 and x < 1 **f.** x < 0 and x > 1
g. x < -1 and x > -3 **h.** x > -1 and x < -3

Practice

Solve each of the inequalities and find the range of values which satisfies them:

- **a.** x - 4 > 8 and x + 3 > 2
- **b.** 3 + x < 2 and 4 - x < 1
- **c.** x - 3 < 4 and x + 5 > 3
- **d.** 2x + 1 > 3 and 3x - 4 < 2
- **e.** 5x -6 > 4 and 3x - 2 < 7
- **f.** 3 - x > 1 and 2 + x > 1

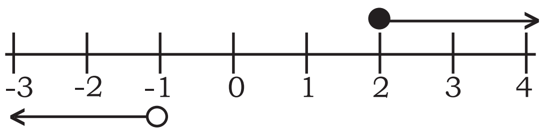
Practice

Find the range of values which satisfy the following inequalities

- **a.** x + 4 < 2x - 1 < 3
- **b.** 3x + 1 < x + 4 < 2
- **c.** 2 - 3x < 4 - x < 3
- **d.** x - 3 < 2x + 1 < 5
- **e.** 4 - 3x < 2x - 5 < 1
- **f.** x < 3x - 1 < x + 1

EXAMPLE: Find the range of values which satisfy the inequalities $x \leq 2$ and $x < -1$.

Draw both inequalities on a number line.



We can see from the number line that there is no range of values which satisfy $x \leq 2$ and $x < -1$

EXAMPLE: Find the range of values which satisfy the inequalities $x - 2 < 2x + 1 < 3$

a) We can write $x - 2 < 2x + 1 < 3$ as two inequalities

$x - 2 < 2x + 1$ and $2x + 1 < 3$

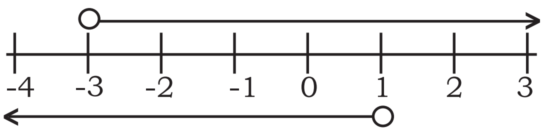
b) Solve both inequalities

$-2 < x + 1$ $2x < 2$

$-3 < x$ $x < 1$

$x > -3$

c) Draw both solutions on a number line



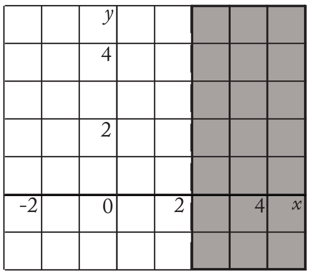
We can see from the number line that $x - 2 < 2x + 1 < 3$ is satisfied in the range: $-3 < x < 1$

6.4 Inequalities and regions

So far we have used number lines to represent inequalities. However, and inequality can also be represented by a **region** on a graph.

EXAMPLE: Draw the region given by the inequality $x < 2$

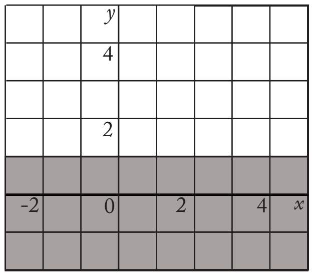
To do this we draw the line $x = 2$. Every point to the left of this line satisfies $x < 2$.



We shade the unwanted part. The line is dotted because $x = 2$ is not included in the region.

EXAMPLE: Draw the region given by the inequality $y \geq 1$

To do this we draw the line $y = 1$. Every point above this line satisfies $y > 1$.



We shade the unwanted part. The line is solid because $y = 1$ is included in the region.

Practice

Draw diagrams to represent the following inequalities:

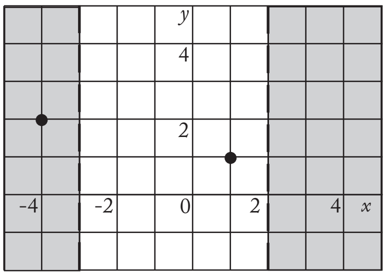
a. x < 2 **b.** y > 3 **c.** x > -1 **d.** y < 4 **e.** x > 0 **f.** 0 > y **g.** 2 < x

EXAMPLE: **a)** Draw the region given by the inequality -3 < x < 2

b) State whether the points (1,1) and (-4,2) lies within the region

-3 < x < 2 gives two inequalities -3 < x and x > 2

The boundary lines of the region are x = - 3 and x = 2. (These lines are both dotted as they are not included in the region)



The unshaded region represents the inequality -3 < x < 2.

We can see that the point (-4,2) does not lie within the region. The point (1,1) does lie within the region.

Practice

i. Draw diagrams to represent the following inequalities:

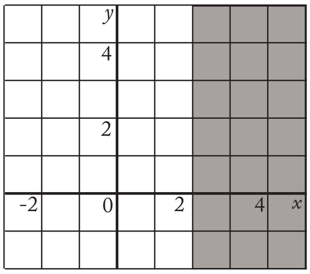
a. $2 < x < 4$ **b.** $-3 < x < 1$ **c.** $-1 < y < 2$ **d.** $0 < x < 4$

e. $-2 < y < 3$ **f.** $3 < x < 5$

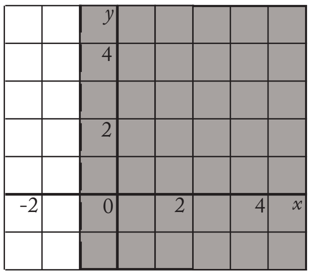
ii. For each question **a.** - **f.** state whether the point (1,4) lies in the given region.

iii. Give the inequalities that define the unshaded regions:

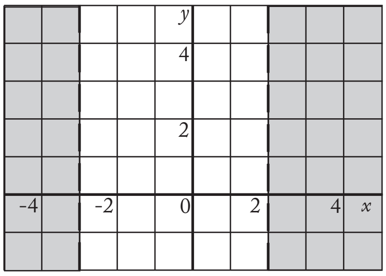
a.



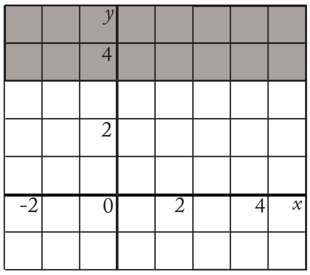
b.



c.



d.

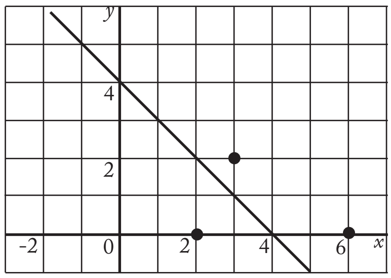


6.5 Inequalities with 2 variables

The **boundary** lines for inequalities are parallel to the x or y axis if the inequality contains only the variable *x* or *y*. If the inequality contains two variables, *x* or *y* then the boundary line is not parallel to an axis.

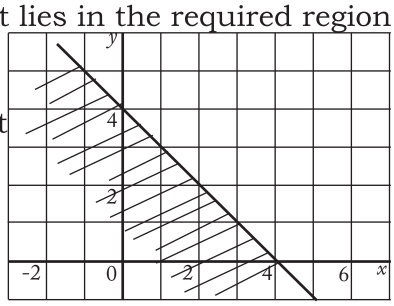
EXAMPLE: Draw the region given by the inequality $x + y \geq 4$

In this example the boundary line is $x + y = 4$.
The line is solid because $x + y = 4$ is included in the region.



We need to decide which side of the line is the region we want.
To do this we test points on both sides of the line.

The point (2,3) gives a value of $x + y = 5$. $5 > 4$ so this point lies in the required region.
The point (5,1) gives a value of $x + y = 6$. $6 > 4$ so this point lies in the required region.
The point (1,1) gives a value of $x + y = 2$. $2 < 4$ so this point does not lie in the required region.
So we shade the region below the line.
The unshaded region gives the region we require.



Practice

Find the regions defined by the following inequalities (draw axes for values of x and y from -6 to 6)

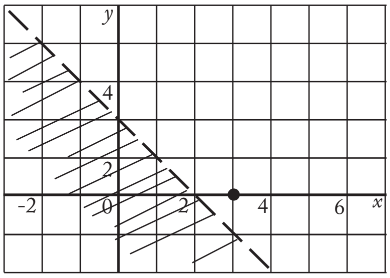
a. $x + y \geq 3$ **b.** $x + 4y \geq 8$ **c.** $x + y < 2$ **d.** $2x + 5y \leq -6$

e. $3x + 4y \geq 12$ **f.** $4x + y < 4$

EXAMPLE: Find the inequality defining the unshaded region.

The boundary line is $x + y = 3$ and is not included.
Test the point (3,1) which is in the required region.

When $x = 3$, $y = 1$. This gives $x + y = 4$.
$4 > 3$, so the inequality is $x + y > 3$

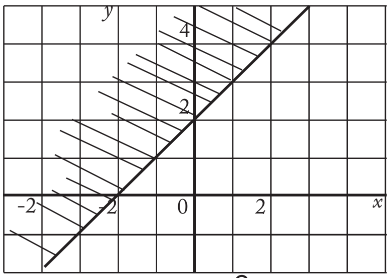


x + y = 3

Practice

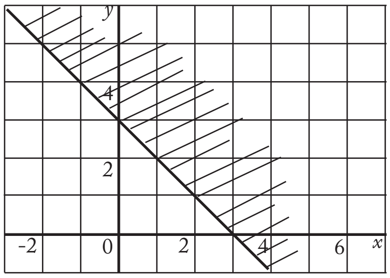
Find the inequalities that define the unshaded regions:

a.



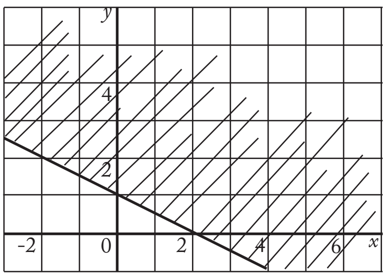
y - x = 2

b.



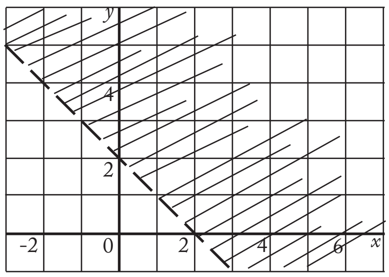
x + y = 3

c.



x + 2y = 2

d.



x + y = 2

7. Quadratic Expressions

7.1 Introduction

In Chapter 1 we studied linear expressions - expressions of degree. In this chapter we will study **quadratic** expressions - expressions of degree 2. These are examples of quadratic expressions:

$$x^2 \qquad z^2 + 3z \qquad 2x^2 + 3x - 1 \qquad a^2 - 1 \qquad p^2 - 4p - 6$$

We can simplify quadratic expressions by collecting like terms and expanding brackets.

EXAMPLE: Simplify $(4x^2 - 7x - 3) - (x^2 - 4x + 1)$

Expand the brackets: $4x^2 - 7x - 3 - x^2 + 4x - 1$

Collect like terms: $3x^2 - 3x - 4$

EXAMPLE: $x(x + 3y) + 2x(x + y)$

Expand the brackets: $x^2 + 3xy + 2x^2 + 2xy$

Collect like terms: $3x^2 + 5xy$

Practice

Simplify

a. $2a^2 - 3b^2 + c + 5a^2 - 6b^2 + c$

b. $(2a^2 - 3b^2 + c) + (5a^2 - 6b^2 + c)$

c. $(a^2 + 2b^2 + 4c^2 - 3d^2) - (3a^2 - 8b^2 - 2c^2 + d^2)$

d. $(4x^2 - 3y^2) + (3y^2 - 5x^2) + (5z^2 - 4x^2)$

e. $4a(2a + b) + 2a(3a - b)$

f. $m(n + m) + m(n - m)$

g. $x(4x + 5) - 2x(x + 3)$

h. $x(2x - 5) - 2(2x - 5)$

i. $5y(4y - 3) - 2(4y - 3)$

j. $3x(x + y) + 4y(x - y)$

Think

Look at question **j.** If we expand the brackets we have:

$$3x(x + y) + 4y(x - y) = 3x^2 + 3xy + 4yx - 4y^2$$

We can simplify the second and third terms:

$$3xy + 4yx = 3xy + 4xy = 7xy$$

Complete the statement:

When multiplying unknowns, the order we multiply is not important.
This is the ________________ law of multiplication.

7.2 Brackets

So far we have only studied the product of a single term and an expression in brackets. We find the product by multiplying each term in the bracket by the single term: $3x(2x + 3 + 2y) = 6x^2 + 9x + 6xy$.
In this section we will learn how to find the product of two brackets.

Look at the rectangle. The area of the whole shape is $(e + f)(g + h)$.

We can use the **FOIL** rule to find the product of these two brackets. **FOIL** stands for:

First - multiply the first terms in the brackets = eg

Outer - multiply the inner terms in the brackets = eh

Inner - multiply the inner terms in the brackets = fg

Last - multiply the last terms in the brackets = fh

If we add the four terms we have: $(e + f)(g + h) = eg + eh + fg + fh$

	g	h
e	area = eg	area = eh
f	area = fg	area = fh

The equation $(e + f)(g + h) = eg + eh + fg + fh$ is true for all values of e, f, g and h. The equation is known as an **identity**. We will meet more identities in this section.

Practice

i. Find the product of these brackets. Draw rectangles to illustrate and check your answers

a. $(x + 2)(x + 1)$ **b.** $(x + 8)(x + 2)$ **c.** $(x + 3)(x + 3)$ **d.** $(x + a)(x + b)$

ii. Use the FOIL rule to find the product of these brackets. Simplify your answers if possible

a. $(a + 4)(b + 3)$ **b.** $(x + 6)(x + 2)$ **c.** $(b + 7)(b + 4)$ **d.** $(2a + 3)(b + 4)$

e. $(3b + 4)(b + 2)$ **f.** $(3b + 4)(b - 2)$ **g.** $(x + 6)(x - 2)$ **h.** $(a - 4)(b - 3)$

i. $(x - 7)(x - 4)$ **j.** $(b - 6)(b - 2)$ **k.** $(a - 4)(b + 3)$ **l.** $(b - 7)(b + 4)$

iii. Use the FOIL rule to find the product of these brackets. Simplify your answers if possible

a. $(3a - 4)(b + 5)$ **b.** $(x + y)(2x + 3y)$ **c.** $(2b + 5c)(3b + 4c)$ **d.** $(a + 3)^2$

e. $(a + 1)(b + 2) - ab$ **f.** $(3x + 1)(3x - 1)$ **g.** $(7x + 2)(7x - 2)$

h. $5(x + 2)(x - 6)$ **i.** $(2a + 3b)(2a - 3b) + 9b^2$

Think

i. There are 3 very important identities that you need to remember. Use the FOIL rule to find them:

$(x + a)^2$ = ____________________ = ____________________ = ____________________

$(x - a)^2$ = ____________________ = ____________________ = ____________________

$(x + a)(x - a)$ = ____________________ = ____________________

ii. Complete the sentences to give the identities in words:

a. $(x + a)^2$

Square of the first term + ________________________ + ________________________

b. $(x - a)^2$

________________________ - twice the product of the two terms + ________________________

Practice

Use the identities to expand

i. a. $(x + 1)^2$ **b.** $(x + 2)^2$ **c.** $(b + 4)^2$ **d.** $(t + 10)^2$ **e.** $(x + y)^2$ **f.** $(p + q)^2$ **g.** $(u + v)^2$

ii. a. $(x - 2)^2$ **b.** $(x - 6)^2$ **c.** $(x - y)^2$ **d.** $(x - 7)^2$ **e.** $(a - b)^2$ **f.** $(3x - 1)^2$ **g.** $(4y - 1)^2$

iii. a. $(x + 4)(x - 4)$ **b.** $(x + 12)(x - 12)$ **c.** $(x - 8)(x + 8)$

d. $(2x + 1)(2x - 1)$ **e.** $(5x - 4)(5x + 4)$

Complete these expressions by writing the correct number in each space

a. (x + ___)2 = x^2 + ___x + 16 **b.** (x - ___)2 = x^2 - 20x + ___ **c.** (x + ___)^2 = x^2 + 2x + ___

d. (x - ___)2 = x^2 - ___x + 144

7.3 Factorising quadratic expressions

We learnt in Section 2.1 that we can factorise expressions by finding the Highest Common Factor of the terms in the expression.

> **EXAMPLE:** Factorise $2a^2 + 6a$
>
> $2a^2 + 6a = \underline{2} \times \underline{a} \times a + \underline{2} \times 3 \times \underline{a}$
>
> The HCF of the two terms is $2a$, so
>
> $2a^2 + 6a = 2a(a + 3a)$

> **EXAMPLE:** Factorise $9a^2b + 12ab^2$
>
> $9a^2b + 12b^2 = \underline{3} \times 3 \times \underline{a} \times a \times \underline{b} + \underline{3} \times 4 \times \underline{a} \times \underline{b} \times b$
>
> The HCF of the three terms is $3ab$, so
>
> $9a^2b + 12b^2 = 3ab(3a + 4b)$

Practice

Factorise

a. $8x^2 + 4x$ **b.** $6p^2 + 3p$ **c.** $15c - 10c^2$ **d.** $ax + ax^2$

e. $12a^2b + 18ab^2$ **f.** $4x^2y - 2xy^2$ **g.** $4a^2b + 8ab^2 + 12ab$

h. $4x^2y + 6xy^2 - 2xy$ **i.** $12ax^2 + 6a^2x - 3ax$ **j.** $a^2bc + ab^2c + abc^2$

In the previous section we learnt that we can find the product of two brackets using the FOIL method, for example $(x + 6)(x + 2) = x^2 + 8x + 12$. To reverse the operation and factorise $x^2 + 8x + 12$, we write:

$$x^2 + 8x + 12 = (x + \quad)(x + \quad)$$

(This will give the x^2 term when we expand.)

Now we need to find two numbers whose product is 12 and whose sum is 8:

12 = 1 x 12 or 3 x 4 or 2 x 6

The sum of 2 and 6 is 8, so these are the two numbers we need. We can factorise the expression:

$$x^2 + 8x + 12 = (x + 2)(x + 6)$$

EXAMPLE: Factorise $x^2 + 8x + 15$

15 = 3 x 5 or 1 x 15. 8 = 3 + 5.

So, $x^2 + 8x + 15 = (x + 3)(x + 5)$

EXAMPLE: Factorise $x^2 - 7x + 12$

The coefficient of the x term is negative, so the two numbers have to be negative.

12 = -3 x -4 or -1 x -12 or -2 x -6.

And, -7 = -3 - 4.

So, $x^2 - 7x + 12 = (x - 3)(x - 4)$

Practice

i. Finish factoring these expressions

a. $x^2 + 6x + 5 = (x + 1)(x + \quad)$
b. $x^2 + 7x + 12 = (x + \quad)(x + 4)$
c. $x^2 + 17x + 52 = (x + 4)(x + \quad)$
d. $x^2 - 9x + 8 = (x - 1)(x \quad)$
e. $x^2 - 11x + 28 = (x - 7)(x \quad)$
f. $x^2 - 18x + 32 = (x - 2)(x \quad)$

ii. Check your answers to **i.** by expanding the brackets

iii. Factorise. Check your answers by expanding the brackets

a. $x^2 + 7x + 12$ **b.** $x^2 + 21x + 20$ **c.** $x^2 + 8x + 7$ **d.** $x^2 + 13x + 12$
e. $x^2 + 15x + 36$ **f.** $x^2 + 22x + 40$ **g.** $x^2 + 14x + 40$ **h.** $x^2 - 9x + 8$
i. $x^2 - 7x + 12$ **j.** $x^2 - 11x + 28$ **k.** $x^2 - 18x + 32$ **l.** $x^2 - 16x + 63$

Think

i. In this expression the number term is negative. Complete the factorisation by writing the correct signs in the brackets.

$$x^2 + 5x - 24 = (x \quad 8)(x \quad 3)$$

ii. Complete the statements

If the x term is positive and the number term is positive then the signs in the brackets are

____________.

If the x term is positive and number term is negative then the signs in the brackets are

______________.

Practice

Factorise. Check your answers by expanding the brackets

a. $x^2 + 5x - 14$ **b.** $x^2 + x - 30$ **c.** $x^2 - 6x - 27$ **d.** $x^2 + 16x - 80$
e. $x^2 + 8x - 48$ **f.** $x^2 - 11x - 42$ **g.** $x^2 + 7x - 60$ **h.** $12x - 28 + x^2$
i. $6x + x^2 - 7x - 42$ **j.** $2x - 63 + x^2$

Sometimes the coefficient of the x^2 term is greater than 1.
In some cases, we can remove a common factor from the expression before expanding.

EXAMPLE: Factorise $3x^2 + 9x + 6$

$3x^2 + 9x + 6 = 3(x^2 + 3x + 2) = 3(x + 1)(x + 2)$

In some cases, one of the x terms in the bracket will have a coeffcient that is greater than 1.

EXAMPLE: Factorise $5x^2 - 7x + 2$

5 = 5 x 1, so to start we write $(5x \quad)(x \quad)$.

The signs in the brackets must be negative $(5x - \quad)(x - \quad)$.

If we write 1 and 2 for the number terms we have $(5x - 1)(x - 2)$.

This gives a middle term of $11x$, which is not correct.

If we try 2 and 1 we have $(5x - 2)(x - 1)$

This gives a middle term of $7x$, which is correct.

So the correct answer is $5x^2 - 7x + 2 = (5x - 2)(x - 1)$

Think

Factorise $15x^2 + 26x + 8$, then:

The factors of 15 are 1, 3, 5 and 15. The first terms of the brackets may be

$(15x \quad)(x \quad)$ or $(5x \quad)(3x \quad)$

The number terms in the factorisation can be 1 and 8, 2 and 4, 4 and 2 or 8 and 1.

We have to find the right numbers so that the middle term in the expansion is equal to 26x.

Complete the gaps in the factorisations below until you get the correct middle term of the expression.

$(15x + 8)(x + 1)$ middle term is 23x which is incorrect.

$(15x + 4)(x + 2)$ middle term is _____ which is __________.

$(15x + 2)(x + \quad)$ middle term is _____ which is __________.

$(15x + 1)(x + \quad)$ middle term is _____ which is __________.

$(5x + \quad)(3x + 8)$ middle term is _____ which is __________.

$(5x + \quad)(3x + 4)$ middle term is _____ which is __________.

$(5x + 8)(3x + \quad)$ middle term is _____ which is __________.

$(5x + 4)(3x + \quad)$ middle term is _____ which is __________.

The final answer is $15x^2 + 26x + 8 =$ ____________________.

In the most general case, both the coefficients of x in the brackets are greater than 1.

Practice

i. Factorise:

a. $2x^2 + 14x + 24$ **b.** $7x^2 + 14x + 7$ **c.** $4x^2 - 4x - 48$ **d.** $3x^2 + 24x + 36$

e. $5x^2 - 5x - 30$

ii. Factorise

a. $2x^2 + 3x + 1$ **b.** $4x^2 + 7x + 3$ **c.** $3x^2 + 13x + 4$ **d.** $4x^2 + 5x - 6$

e. $3x^2 - 8x + 4$ **f.** $5x^2 - 17x + 6$ **g.** $7x^2 - 29x + 4$ **h.** $5x^2 - 19x + 12$

i. $4x^2 + 17x - 15$ **j.** $3x^2 - 11x - 20$

iii. Factorise

a. $6x^2 + 7x + 2$ **b.** $15x^2 + 11x + 2$ **c.** $35x^2 + 24x + 4$ **d.** $15x^2 - x - 2$

e. $24x^2 + 17x - 20$ **f.** $6x^2 - 11x + 3$ **g.** $16x^2 - 10x + 1$ **h.** $15x^2 - 44x + 21$

i. $6a^2 - a - 15$ **j.** $9b^2 - 12b + 4$

The difference of two squares

Earlier we saw that $(x + a)(x - a) = x^2 - a^2$. If we reverse this we have

$$x^2 - a^2 = (x + a)(x - a)$$

This result is called the **difference of two squares**. It is used often in Maths. We can use it to factorise expressions.

Practice

a. $x^2 - 25$ **b.** $x^2 - 4$
c. $x^2 - 64$ **d.** $x^2 - 49$
e. $9 - x^2$ **f.** $36 - x^2$
g. $25 - x^2$ **h.** $y^2 - x^2$

> **EXAMPLE:** Factorise $x^2 - 9$
> We start with $(x \quad)(x \quad)$
> $-9 = 3 \times -3$, so we have, $x^2 - 9 = (x + 3)(x - 3)$

7.4 Algebraic fractions

In this section we will combine knowledge of fractions, factorising and completing the square to simplify algebraic fractions with quadratic terms.

Think

Try to follow the steps to simplify the following expression.

$$\frac{2x - 4}{x^2 - 5x + 6} \div \frac{3x + 9}{x^2 - 9}$$

Factorise all the terms ______________________________

Cancel the common factors ______________________________

Invert the second fraction ______________________________

Cancel the common factors ______________________________

> **EXAMPLE:** Simplify $\frac{6x - 30}{2x^2 - 50} + \frac{21}{x^2 + 3x - 10}$
>
> a) Factorise the terms $\frac{6(x - 5)}{2(x - 5)(x + 5)} + \frac{21}{(x + 5)(x - 2)}$
>
> b) Cancel by $2(x - 5)$ $\frac{3}{x + 5} + \frac{21}{(x + 5)(x - 2)}$
>
> c) Write as one fraction with a common denominator $\frac{3(x - 2) + 21}{(x + 5)(x - 2)}$
>
> d) Simplify the numerator $\frac{3x + 15}{(x + 5)(x - 2)}$
>
> e) Factorise the numerator $\frac{3(x + 5)}{(x + 5)(x - 2)}$
>
> f) Cancel $(x + 5)$ to give the answer $\frac{3}{(x - 2)}$

Practice

Simplify

a. $\frac{x^2 - 4}{2x - 4}$ **b.** $\frac{x^2 - 5x + 4}{x^2 - 16}$ **c.** $\frac{3}{x + 1} + \frac{2}{x + 1}$ **d.** $\frac{3}{x + 5} - \frac{2}{x^2 - 25}$ **e.** $\frac{5}{x^2 - 49} - \frac{9}{x - 7}$

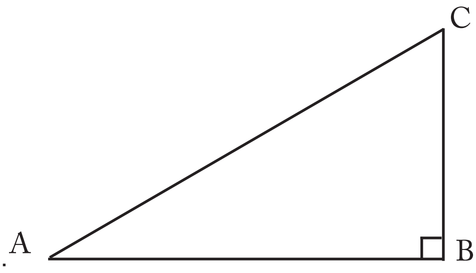
f. $\frac{3}{x^2 + 5x + 4} + \frac{1}{x + 4}$ **g.** $\frac{2}{x^2 + 4x + 3} + \frac{1}{x^2 + 5x + 6}$ **h.** $\frac{4x + 8}{4x^2 - 8x} \div \frac{x^2 - 4}{x^2 + 2x}$

i. $\frac{x^2 - 5x + 4}{x^2 - 16} + \frac{x^2 + 11x + 18}{x^2 + 6x + 8}$

8. Pythagoras' Theorem

8.1 Introduction

Pythagoras' theorem states that, in a right-angled triangle, the square of the longest side (the **hypotenuse**) is equal to the sum of the squares of the other two sides, i.e.



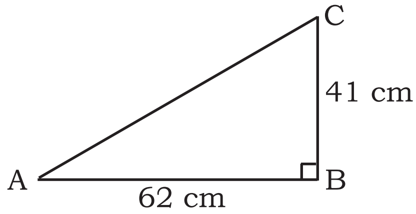
$$AC^2 = AB^2 + BC^2$$

If we know the length of two sides, we can use Pythagoras' theorem to calculate the length of the third side.

EXAMPLE: Find the length of side AC.

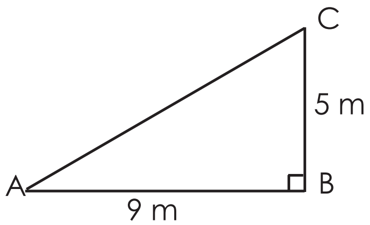
$AC^2 = AB^2 + BC^2$
$= 62^2 + 41^2$
$= 3844 + 1681$
$= 5525$

If we take the square root, we have
$AC = \sqrt{5525} = 74.33$ cm

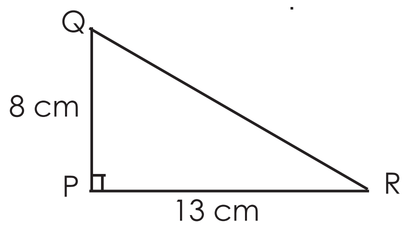


Practice

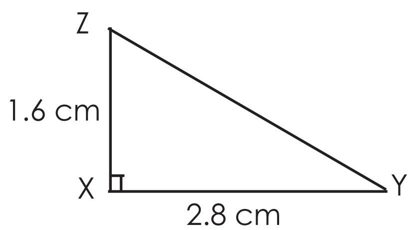
a. Find AC



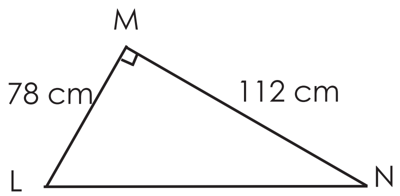
b. Find RQ



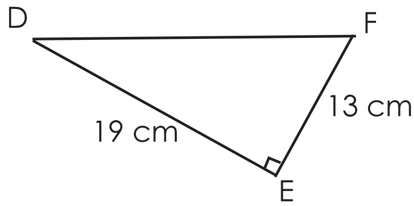
c. Find ZY



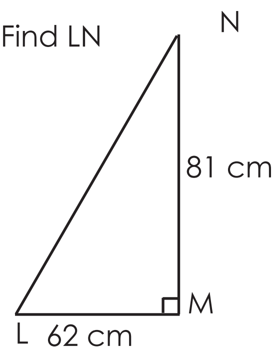
d. Find LN



e. Find DF



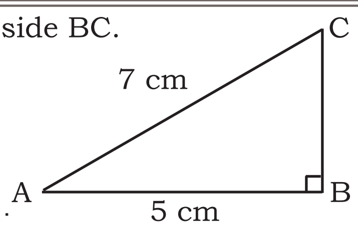
f. Find LN



We can also use Pythagoras' theorem to find the length of one of the shorter sides.

EXAMPLE: Find the length of side BC.

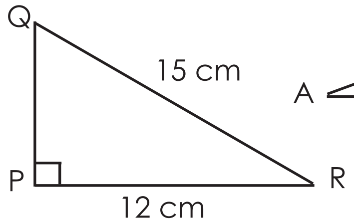
$AC^2 = AB^2 + BC^2$
$7^2 = 5^2 + BC^2$
So,
$BC^2 = 7^2 - 5^2 = 49 - 25 = 24$



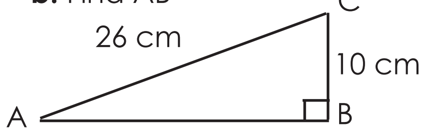
If we take the square root, we have
$BC = \sqrt{24} = 4.9$ cm

Practice

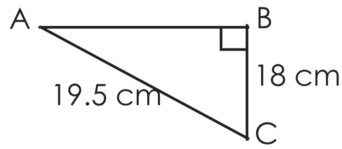
a. Find PQ



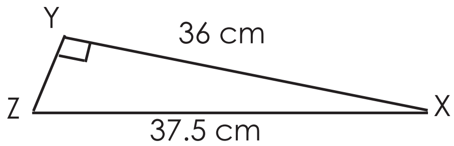
b. Find AB



c. Find AB



d. Find YZ

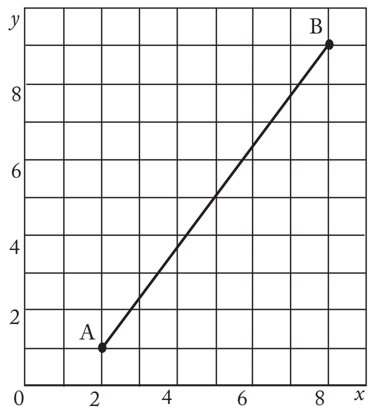


8.2 Using Pythagoras' theorem

We can use Pythagoras' theorem to solve problems.

EXAMPLE: A is the point (2,1). B is the point (8,9)

Find the straight line distance AB



To do this we make a triangle ABM. The angle at M is 90°. The line AM is 6 units and the line BM is 8 units

Pythagoras' theorem gives

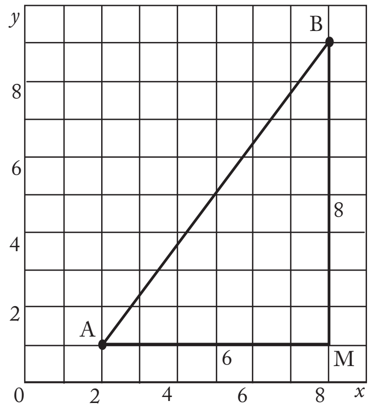
$AB^2 = AM^2 + BM^2$

$= 6^2 + 8^2$

$= 36 + 64$

$= 100$

So, AB = 10

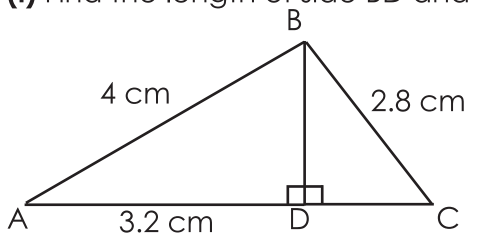


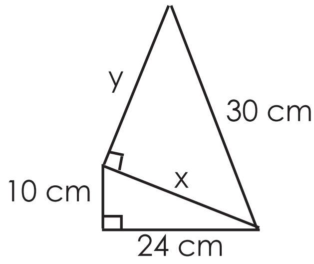
Practice

a. A is the point (3,1) and B is the point (7,9). Find the length of AB.

b. Find the length of the diagonal of a square of side 15 cm.

c. **(i)** Find the length of side BD and **(ii)** Find the length of side DC.

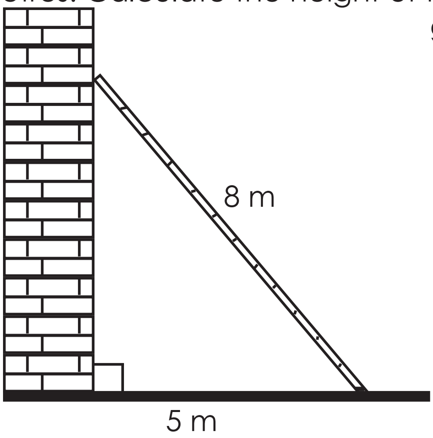




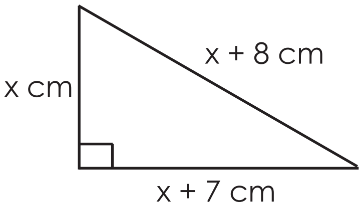
d. Calculate the lengths marked x and y.

e. Points A and B have coordinates (-3, 7) and (5, -4) respectively. Calculate the distance AB.

f. The diagram shows a ladder of length 8 metres resting against a wall. The distance between the bottom of the ladder and the wall is 5 metres. Calculate the height of the top of the ladder.



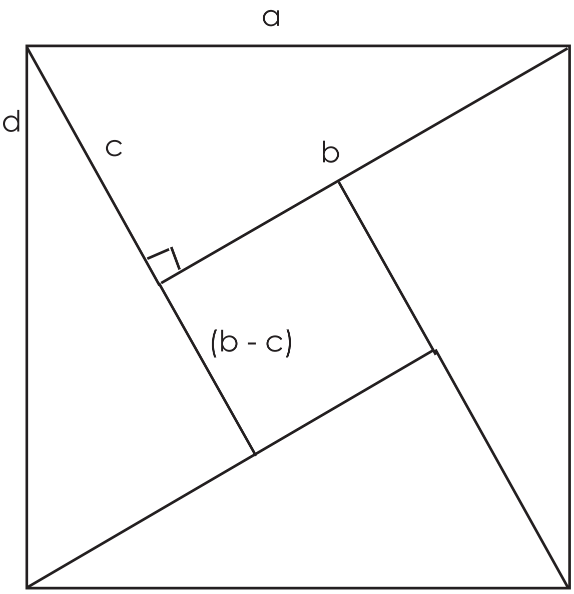
g. Calculate the value of x.



8.3 Proof of Pythagoras' theorem

Think

By looking at the diagram we can see that:
The area of the large square is a^2
The area of the small square is $(b - c)^2$ and
The area of each of the four triangles is ½ bc.
This gives us six formulae. Arrange them below to find an expression for a^2 in terms of b^2 and c^2.



__________ = __________ + __________

= ________________ + __________

= _____ + _____

This exercise is one of many ways to prove that Pythagoras' Theorem is true.

9. Quadratic Equations

9.1 Introduction

In this chapter we will study how to solve quadratic equations of the form $ax^2 + bx + c = 0$.
We will learn four methods to solve quadratic equations:

1. Solving by factorising,

2. Solving by completing the square,

3. The quadratic formula,

4. Solving by drawing graphs.

Equations of this form always have two solutions. There are two values of x which *satisfy* the equation.

Think

When solving quadratic equations, always remember that X x Y = 0 is only true if X = 0 or/and Y = 0. Use this fact to fill the gaps.

If we let X = (x - a) and Y = (x - b) then we have the quadratic equation: (x - a)(x - b) = 0.

This quadratic equation is *only* true if: __________ = 0 or/and ___________= 0.

Or if: x = _____ or x = ______.

The final statement gives the two values of x which satisfy the equation (x - a)(x - b) = 0.

EXAMPLE: Which values of x satisfy $x(2x - 4) = 0$.

Either: $x = 0$ or/and $2x - 4 = 0$
$x = 0$ or/and $x = 2$

EXAMPLE: Which values of x satisfy $(x - 3)(x + 5) = 0$.

Either: $x - 3 = 0$ or/and $x + 5 = 0$
$x = 3$ or/and $x = -5$

Practice

Find the values of x which satisfy the equations:

i. a. $x(x - 3) = 0$ **b.** $x(x - 5) = 0$ **c.** $x(x + 4) = 0$ **d.** $(x + 5)x = 0$
e. $(x - 7)x = 0$ **f.** $(x + 9)x = 0$

ii. a. $(x - 1)(x - 2) = 0$ **b.** $(x - 10)(x - 7) = 0$ **c.** $(x - 6)(x - 1) = 0$
d. $(x + 1)(x + 8) = 0$ **e.** $(x - a)(x - b) = 0$

iii. a. $(2x - 5)(x - 1) = 0$ **b.** $(5x - 4)(4x - 3) = 0$ **c.** $x(10x - 3) = 0$
d. $(6x + 5)(3x - 2) = 0$ **e.** $(4x + 3)(2x + 3) = 0$

9.2 Solving by factorising

If we understand how to factorise and can answer the questions above, then this method is very easy.

EXAMPLE: Solve $x^2 - 10x + 9 = 0$

The coefficient of the x term is negative.
9 = -1 x -9 or -3 x -3
And, -10 = -1 - 9.
So, $x^2 - 10x + 9 = (x - 1)(x - 9) = 0$
The solutions to the equation are:
$x = 1$ or/and $x = 9$

EXAMPLE: Solve $x^2 - 49 = 0$

Start with $(x \quad)(x \quad)$
-49 = -7 x 7
This will give a 0 x term, which is correct.
So, $x^2 - 49 = (x - 7)(x + 7) = 0$
The solutions to the equation are:
$x = 7$ or/and $x = -7$

Practice

i. Solve these equations by factorising:

a. $x^2 - 3x + 2 = 0$ **b.** $x^2 - 5x + 6 = 0$ **c.** $x^2 - 7x + 12 = 0$ **d.** $x^2 + 6x - 7 = 0$
e. $x^2 + x - 12 = 0$ **f.** $x^2 - 5x - 24 = 0$ **g.** $x^2 + 3x + 2 = 0$ **h.** $x^2 + 9x + 18 = 0$
i. $x^2 + 14x + 13 = 0$ **j.** $x^2 + 16x + 15 = 0$

ii. Complete the square to solve these equations

a. $x^2 - 1$ **b.** $x^2 - 16$ **c.** $x^2 - 4$ **d.** $x^2 - 49$ **e.** $x^2 - 81$ **f.** $x^2 - 144$

iii. Look back at Section 5.3 to review how to factorise quadratic expressions with an x^2 coefficient greater than 1. Use the method to solve these equations

a. $2x^2 - 5x + 2 = 0$ **b.** $3x^2 + 5x + 2 = 0$ **c.** $3x^2 - 11x + 6 = 0$
d. $15x^2 + 14x - 8 = 0$ **e.** $6x^2 - 13x - 5 = 0$ **f.** $8x^2 - 18x + 9 = 0$
g. $4x^2 + 8x + 3 = 0$ **h.** $10x^2 - 29x - 21 = 0$

iv. To solve these equations you need to write them in the form $ax^2 + bx + c = 0$ and then factorise

a. $x(x - 2) = 15$ **b.** $x(x + 1) = 12$ **c.** $3x(2x + 1) = 4x + 1$ **d.** $5x(x - 1) = 4x^2 - 4$
e. $x(x + 3) = 5(3x - 7)$

Sometimes a fractional equation leads to a quadratic equation which will factorise.

EXAMPLE: Solve the equation $\frac{6}{x} + \frac{1}{x - 5} = 2$

a) Write as one fraction with a common denominator

$\frac{6(x - 5) + x}{x(x - 5)} = 2$

b) Multiply by $x(x - 5)$ to get $6(x - 5) + x = 2 \text{ x } x(x - 5)$
c) Expand the brackets: $6x - 30 + x = 2x - 10x$
d) Write in the form $ax + bx + c = 0$: $2x^2 - 17x + 30 = 0$
e) Factorise: $(2x - 5)(x - 6) = 0$
f) Solve $2x - 5 = 0$ or $x - 6 = 0$, $x = 2.5$ or $x = 6$

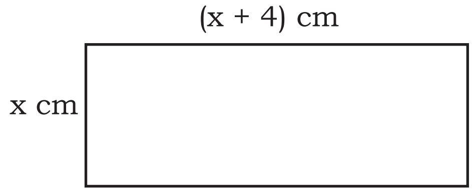
Practice

Write each equation as a quadratic equation and then solve.

a. $\frac{5}{x+3} - \frac{1}{x} = \frac{1}{2}$ **b.** $\frac{2}{x+4} - \frac{1}{x} = 1$ **c.** $\frac{2}{x+1} + \frac{3}{x+4} = \frac{2}{3}$ **d.** $\frac{1}{x-3} - \frac{3}{x-4} = \frac{1}{2}$

e. $\frac{3}{5-x} - \frac{3}{4} = \frac{1}{x+2}$

EXAMPLE: A rectangle is 4 cm longer than it is wide. It is *x* cm wide and has an area of 77 cm^2. Form an equation in *x* and solve it to find the length and width of the rectangle.



Area = length x breadth = $(x + 4) \times x$ cm^2.
We know the area is 77 cm^2. So,

$$(x + 4) \times x = 77$$
$$x^2 + 4x = 77$$
$$x^2 + 4x - 77 = 0$$
$$(x - 7)(x + 11) = 0$$
$$x = 7 \text{ or } x = -11$$

The breadth of the rectangle has to be positive, so we use $x = 7$.
Therefore the breadth of the rectangle is 7 cm and the length is (7 + 4) cm = 11 cm.

EXAMPLE: Tii Moo is *x* years old and his sister is 5 years older. The product of their ages is 84. Form an equation in *x* and solve it to find Peter's age.

Tii Moo is *x* years old. His sister is $(x + 5)$ years old. The product of their ages is 84. So,

$$x(x + 5) = 84$$
$$x^2 + 5x = 84$$
$$x^2 + 5x - 84 = 0$$
$$(x - 7)(x + 12) = 0$$
$$x = 7 \text{ or } x = -11$$

Tii Moo's age must be positive, so Tii Moo is 7 years old.

Practice

These word problems lead to equations that factorise. Write the equation for each problem and solve

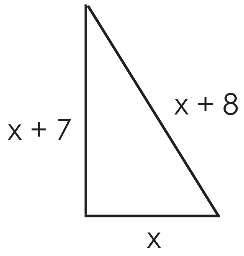
a. Soo Doe is x years old and her sister Aye Phon is 4 years younger. The product of their ages is 140. Form an equation in x and solve it to find Soo Doe and Aye Phon's ages.

b. A rectangle is 3 cm longer than it is wide. It is x cm wide and has an area of 77 cm^2. Form an equation in x and solve it to find the length and width of the rectangle.

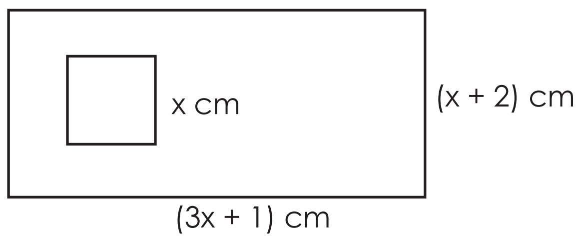
c. A rectangle is 5 cm longer than it is wide. It is x cm wide and has an area of 66 cm^2. Form an equation in x and solve it to find the length and width of the rectangle.

d. One side of a rectangle is 4 cm longer than the other. Find the length of the sides if the area of the rectangle is 45 cm 2.

e. The sides of a right-angled triangle are x, (x + 7) and (x + 8). Use a famous mathematical formula to find x and the length of each side.

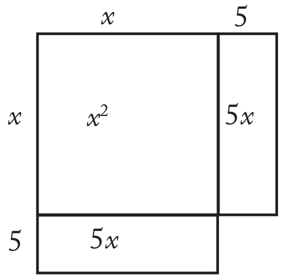


f. A square of side x cm is removed from a rectangle measuring (3x + 1) cm by (x + 2) cm. The area of the remaining card is 62 cm^2. Write an equation in x and solve it to find the length of the sides of the rectangle.

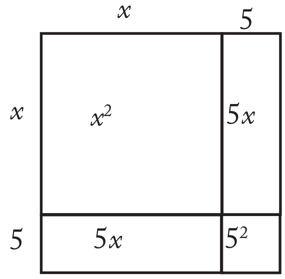


9.3 Completing the square

The equation $x^2 + 10x$ can be shown as a diagram. x^2 is the large square, then we add $10x$ in two equal pieces of 5x.



If we add a small piece to the diagram we have a perfect square. The area of the small square is $5^2 = 25$



If we add the 52 to the equation we have: $x^2 + 10x + 5^2$

Factorising gives: $x^2 + 10x + 5^2 = (x + 5)(x + 5) = (x + 5)^2$

Subtracting 52 from both sides gives: $x^2 + 10x = (x + 5)^2 - 5^2$

In this example we have **completed the square** for $x^2 + 10x$.
In general to complete the square we use the formula:

$$x^2 + bx = \left[x + \frac{b}{2}\right]^2 - \left[\frac{b}{2}\right]^2$$

We can only use this if the coefficient of x^2 is equal to 1.

EXAMPLE: Complete the square for $x^2 + 4x$

In this example $b = 4$, so we can substitute into the formula to get:

$$x^2 + 4x = \left[x + \frac{4}{2}\right]^2 - \left[\frac{4}{2}\right]^2$$

EXAMPLE: Complete the square for $2x^2 - 12x$

Before we use the formula we take out the common factor: $2x^2 - 12x = 2[x^2 - 6x]$.
Now we can substitute the expression in brackets into the formula to get:

$$2x^2 - 12x = 2\left[\left(x + \frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2\right] = 2[(x - 3)^2 - 9]$$

Practice

i. Complete the square for the following expressions:

a. $x^2 + 4x$ **b.** $x^2 - 14x$ **c.** $x^2 + x$ **d.** $x^2 - 4x$ **e.** $x^2 - 10x$

ii. Complete the square for the following expressions. Take out the common factor first

a. $2x^2 + 16x$ **b.** $3x^2 - 12x$ **c.** $5x^2 - 15x$ **d.** $2x^2 + 2x$ **e.** $7x^2 - 28x$

9.4 Solving by completing the square

In Section 7.2 we learnt how to solve equations by factorising. Some equations cannot be easily factorised. We can solve these equations by completing the square.

Think

Follow the steps to solve the equation $x^2 + 10x + 18 = 0$ by completing the square.

Subtract 18 from both sides: ____________________

Complete the square for $x^2 + 10x$ (see previous page): ____________________

Add 25 to both sides: ____________________

Square root both sides: ____________________

Subtract 5 from both sides: ____________________

EXAMPLE: Solve the equation $x^2 + 5x - 3 = 0$ by completing the square. Leave your answer in surd form.

Add 3 to both sides: $x^2 + 5x = 3$

Complete the square using the formula: $\left[x + \frac{5}{2}\right]^2 - \left[\frac{5}{2}\right]^2 = 3$

$\left[\frac{5}{2}\right]^2 = \frac{25}{4}$. Add this to both sides: $\left[x + \frac{5}{2}\right]^2 = \frac{37}{4}$

Take the square root of both sides: $x + \frac{5}{2} = \pm\sqrt{\frac{37}{4}} = \pm\sqrt{\frac{37}{2}}$

Subtract $\frac{5}{2}$ from both sides to solve: $x = -\frac{5}{2} = \pm\sqrt{\frac{37}{2}}$

Note: In both examples we are asked to leave the answer in **surd form**. This means we do not calculate the value of the square root. You will usually be told which form to write your answer, e.g. surd form, 2 decimal places, 3 significant figures etc.

Practice

Solve the equations by completing the square. Leave your answer in surd form

a. $x^2 + 10x + 3 = 0$ **b.** $x^2 - 8x - 2 = 0$ **c.** $2x^2 + 18x + 6 = 0$ **d.** $3x^2 - 6x + 1 = 0$

e. $2x^2 - 3x - 4 = 0$ **f.** $2x^2 - 4x - 7 = 0$ **g.** $3x^2 + 5x - 3 = 0$ **h.** $2x^2 - 7x - 2 = 0$

i. $3x^2 - 8x - 1 = 0$

9.5 The quadratic formula

We can apply the method of completing the square to solve the general equation $ax^2 + bx + c = 0$. The solution will give us a formula which can be used to solve all quadratic equations. Follow the steps of the solution.

Start with the general equation	$ax^2 + bx + c = 0$
Divide both sides by a	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
Subtract $\frac{c}{a}$ from both sides	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
Add $\frac{b^2}{4a^2}$ to both sides to complete the square	$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a}x + \frac{b^2}{4a^2}$
Factorise the left side of the equation	$\left[x^2 + \frac{b}{2a}\right]^2 = -\frac{c}{a}x + \frac{b^2}{4a^2}$
Simplify the right side of the equation	$\left[x^2 + \frac{b}{2a}\right]^2 = \frac{-4ac + b^2}{4a^2}$
Take the square root of both sides	$x^2 + \frac{b}{2a} = \sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)} = \pm\frac{\sqrt{(b^2 - 4ac)}}{2a}$
Subtract $\frac{b}{2a}$ from each side	$x = -\frac{b}{2a} \pm \frac{\sqrt{(b^2 - 4ac)}}{2a}$

$$x = -\frac{b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

The two solutions of the equation are $x = -\frac{b}{2a} + \frac{\sqrt{(b^2 - 4ac)}}{2a}$ and $x = -\frac{b}{2a} - \frac{\sqrt{(b^2 - 4ac)}}{2a}$

Practice

i. Use the formula to solve these equations:

a. $x^2 + 6x + 5 = 0$
b. $x^2 + 5x + 4 = 0$
c. $x^2 + 9x + 8 = 0$
d. $x^2 + 10x - 11 = 0$
e. $x^2 + 9x - 10 = 0$
f. $x^2 - 7x + 10 = 0$
g. $x^2 - 7x - 18 = 0$
h. $x^2 - 4x - 12 = 0$
i. $x^2 + 8x + 12 = 0$

ii. Use the formula to solve these equations. Leave your answers in surd form.

a. $2x^2 + 7x + 2 = 0$
b. $4x^2 + 7x + 1 = 0$
c. $5x^2 + 9x + 2 = 0$
d. $2x^2 - 7x + 4 = 0$
e. $3x^2 + 9x + 1 = 0$

EXAMPLE: Solve the equation $3x^2 + 7x - 2 = 0$.
Leave your answer in surd form.
The coefficients are a = 3, b = 7, c = -2

$x = \frac{-b \pm\sqrt{(b^2 - 4ac)}}{2a}$

Substitute the values of a, b and c

$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-2)}}{2 \times 3}$

$x = \frac{-7 \pm\sqrt{(49 + 24}}{6} = \frac{-7 +\sqrt{73}}{6}$

In surd form the two solutions are

$x = \frac{-7 + \sqrt{73}}{6}$ or $x = \frac{-7 - \sqrt{73}}{6}$

iii. Rearrange these equations and use the formula to solve. Leave your answers in surd form.

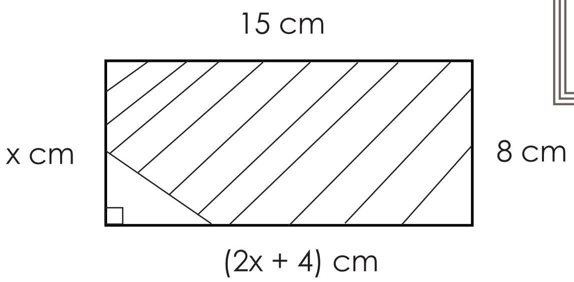
a. $3x^2 = 3 - 5x$
b. $3x^2 + 2 = 9x$
c. $8x^2 = x + 1$
d. $3x^2 + 4x = 1$

iv. Use the quadratic formula to solve these problems

a. One side of a rectangle is 3 cm longer than the other. Find the length of the sides if the area is 20 cm^2. (Use the fact that $\sqrt{89} = 9.4$ to 1 decimal place.)

b. The sides of a right-angled triangle are x cm, (x + 1) cm and (x + 3) cm. Find the length of the hypotenuse. (Use the fact that $\sqrt{48} = 6.9$ to 1 decimal place.)

c. A right angled triangle is cut from a rectangular piece of card 15 cm by 8 cm. The remaining piece of card (the shaded area) has an area of 89 cm^2. Find the value of x. (Use the fact that $\sqrt{32} = 5.7$ to 1 decimal place.)



EXAMPLE: Solve the equation $4x^2 - 7x + 1 = 0$.

Give your answer to 2 decimal points.
The coefficients are a = 4, b = -7, c = 1

$$x = \frac{-b \pm\sqrt{(b^2 - 4ac)}}{2a}$$

Substitute the values of a, b and c

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-1)}}{2 \times 4}$$

$$x = \frac{-7 \pm\sqrt{(49 + 16}}{8} = \frac{-7 \pm\sqrt{65}}{8} = \frac{7 \pm 8.062}{8}$$

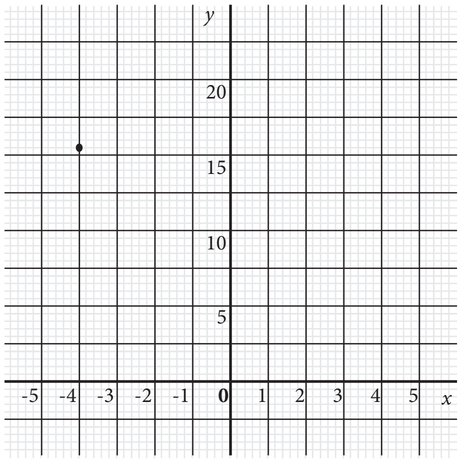
In decimal form the two solutions are

$$x = \frac{15.062}{8} \text{ or } x = \frac{-1.062}{8}$$

or x = 1.88 or x = -0.13, correct to 2 d.p.

9.6 Graphing quadratic equations

In Chapter 3 we learnt how to draw the graph of a linear equation. Here we will learn to draw graphs of quadratic equations. The graph of a quadratic equation is called a **parabola.** The simplest parabola has the equation $y = x^2$.



Think

Follow the steps below to draw the graph of $y = x^2$

a. Complete the table of values by substituting the values of x into the equation.

x	-4	-3	-2	-1	0	1	2	3	4
y	16					1			

b. Plot the points on the axis to the right.

c. Connect all the points with a smooth curve.

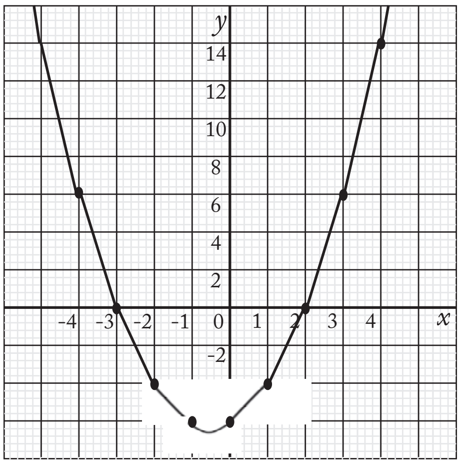
The curve gives the parabola of $y = x^2$

EXAMPLE: Draw the graph of $y = x^2 + x - 6$ for values of x from -4 to 4

a) First we write the table of values for the given range of x.

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
x	-4	-3	-2	-1	0	1	2	3	4
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
$x^2 + x - 6$	6	0	4	-6	-6	-4	0	6	14

b) Next we plot the points on the axis and draw a smooth curve through the points.



Practice

Draw graphs of the following equations for values of x from -4 to 4. (You will need graph paper)

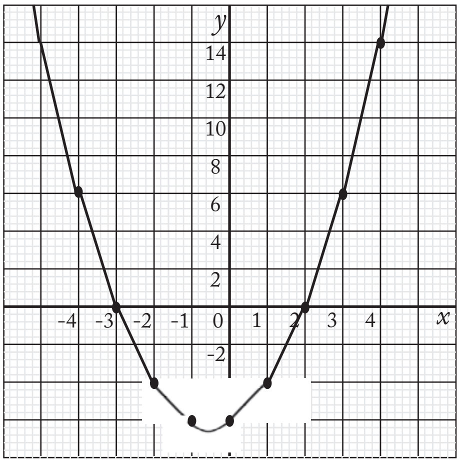
a. $y = -x^2$ **b.** $y = x^2 + 5$ **c.** $y = 3x^2$ **d.** $y = x^2 - 10$

e. $y = x^2 + 2x$ **f.** $y = x^2 - 4x - 1$ **g.** $y = x^2 - 3x - 5$ **h.** $y = 2x^2 + 3x - 5$

9.7 Solving quadratic equations by graphing

Once we have drawn the graph of a quadratic equation, we can use it to find the solutions of the equation.

EXAMPLE: Draw a graph to find the solutions of $x^2 + x - 6 = 0$.



We draw the graph of $y = x^2 + x - 6$.

The equation is $x^2 + x - 6 = 0$ so we look at the line $y = 0$.
The line $y = 0$ is the x-axis.

The solutions are given by the points where the graph crosses the x-axis.

We see from the graph that the solutions are $x = -3$ and $x = 2$

The solutions of a quadratic equation are the values of x where the graph crosses the x-axis.

Practice

Solve the following equations by drawing graphs. Give your answers to 1 d.p. Use the range of values of x that are given in the brackets.

a. $x^2 - 7x + 8 = 0$ (for x from 0 to 8)

b. $x^2 - x - 3 = 0$ (for x from -3 to 4)

c. $2x^2 - 3x - 7 = 0$ (for x from -3 to 4)

d. $5x^2 - 8x + 2 = 0$ (for x from -1 to 3)

10. Sequences

10.1 Introduction

A **sequence** is a set of numbers arranged in an order. The easiest sequence is the sequence of counting numbers:

1, 2, 3, 4, 5, 6, 7,..........

Other easy sequences are:

Odd numbers: 1, 3, 5, 7, 9, 11,.........
Prime numbers: 2, 3, 5, 7, 11, 13
Negative numbers: -1, -2, -3, -4, -5, -6

To continue a sequence we need to look for a **pattern.** For example, look at the sequence:

2, 5, 8, 11,.....

Each term in this sequence is 3 bigger that the previous term. So the sequence continues with:

14, 17, 20,....

Practice

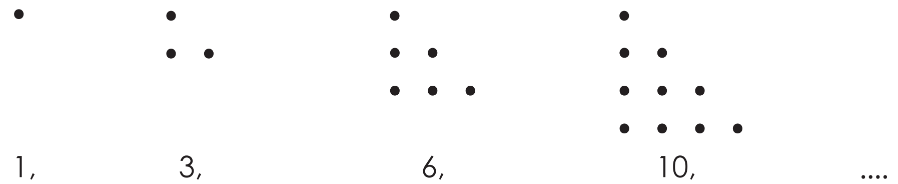
i. For each sequence, find the rule for continuing the sequence and give the next two terms.

a. 1, 4, 9, 16,... **b.** 3, 6, 9, 12,... **c.** 7, 14, 21, 28,... **d.** 15, 11, 7, 3,...
e. 2, 1, 0.5, 0.25,... **f.** 2, 6, 12, 20,...

ii. Fill the gaps in the each sequence

a. 3, 9, __, 21, 27,__,... **b.** 3,__, 48, 192, 768,...

iii. Triangular numbers can be represented by dots arranged in triangles:



Write down the next four numbers in this sequence.

iv. Below is a sequence of tile patterns. Write down the first five terms of the sequence given by the number of tiles in each pattern.

10.2 The nth term of a sequence

Look at the sequence: 1, 3, 5, 7, 9, 11, 13, 15, ...
The 1st term is 1, the 3rd term is 5 and the 6th term is 11.

The general term of a sequence is called the nth term. If we know the formula for the nth term of a sequence, then we can find any term in that sequence.

EXAMPLE: The n th term of a sequence is given by the formula: n th term = $n(n + 1)$
Find the 1st, 2nd and 6th terms.
When: $n = 1$ 1st term = $1(1 + 1) = 2$
$n = 2$ 2nd term = $2(2 + 1) = 6$
$n = 6$ 6th term = $6(6 + 1) = 42$

EXAMPLE: The n th term of a sequence is given by the formula: n th term = $(n - 1)^2$
Find the 1st, 2nd and 6th terms.
When: $n = 1$ 1st term = $(1 - 1)^2 = 0$
$n = 2$ 2nd term = $(2 - 1)^2 = 1$
$n = 6$ 6th term = $(6 - 1)^2 = 25$

i. Find the first four terms and the 7th term for each of the sequences below

a. $2n + 1$ **b.** $2n - 1$ **c.** 2^n **d.** $(n - 1)(n + 1)$ **e.** $n + 4$ **f.** $3 + 2n$

ii. Find the first five terms for each of the sequences below

a. $3n + 7$ **b.** $15 - 2n$ **c.** $n^2 + 5$ **d.** $\left(\frac{1}{2}\right)^n$ **e.** $\frac{n}{2}(n + 1)$ **f.** $\frac{n}{n + 1}$

10.3 Finding the formula for a sequence

When the pattern in a sequence is known, we can use it to find a formula for the *n*th term.

Consider the sequence: 2, 4, 6, 8, 10
We draw a table for *n*, the position of the term in the sequence, and the *n*th term, the value of that term.

n	1	3	4	3	5
*n*th term	2	4	6	8	10

The pattern here is that the value of the term is twice its position number, *n*. So,

$n = 6$ 6th term = 12
$n = 10$ 10th term = 20
$n = n$ n th term = $2n$

The formula for the n th term is n th term = $2n$.

EXAMPLE: Find the formula for the n th term of the sequence 1, 4, 7, 10, ...
Draw the table of values:

n	1	2	3	4	5
*n*th term	1	4	7	10	13

The terms increase by 3 each time, so multiples of $3n$ will be part of the formula

n	1	2	3	4	5
3*n*	3	6	9	12	15

In this table the values of the terms are 2 greater than the terms in the original sequence, so we subtract 2 from $3n$ to get: nth term = $3n - 2$
(We can check our answer by substituting values of n: $n = 1$; 1st term = $3 \times 1 - 2 = 1$, $n = 2$; 2nd term = $3 \times 2 - 2 = 4$, $n = 3$; 3rd term = $3 \times 3 - 2 = 7$. These are the correct values, so we know our formula is correct.)

Practice

Find the formula for the nth term in each sequence.

a. 3, 6, 9, 12,.... **b.** -1, -2, -3, -4,.... **c.** 2, 3, 4, 5,.... **d.** 4, 8, 12, 16,.....

e. 7, 9, 11, 13,..... **f.** 0, 3, 6, 9, 12.... **g.** $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$,.... **h.** 1 x 3, 2 x 4, 3 x 5,.....

i. 1, 8, 27, 64,..... **j.** 3, 2, 1, 0, -1....

11. Indices

11.1 Introduction

In Module 1 we learnt that: $3 \times 3 \times 3 \times 3 = 3^4 = 81$

$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

32 written in **index form** is 2^5. The number 5 is called the **index**. The plural of index is **indices**.
In general we can write: $a \times a \times a \times .... \times a = a^n$

Practice

i. Find the values of these powers:

a. 2^6 **b.** 10^5 **c.** 5^4 **d.** 7^3 **e.** 12^4

ii. Write these numbers in index form:

a. 1000 **b.** 125 **c.** 512 **d.** 625

iii. Show that 64 can be written as either 2^6 or 4^3.

11.2 The index laws

In Module 1 we also learnt about multiplication and division with indices, numbers to the power of 0 and negative powers. Here is a review.

Multiplication

$3^2 \times 3^2 = 3^2+3 = 35$

To multiply 2 powers of the same number, we add the indices. In general:

$$a^m \times a^n = a^{m+n}$$

Numbers to the power of 0

Notice that $1 = \frac{8}{8} = \frac{2^3}{2^3} = 2^{3-3} = 2^0$

We can replace 2 with any other number, to get the general result:

$$a^0 = 1$$

Division

$2^5 \div 2^3 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 = 22$

To divide 2 powers of the same number, we subtract the indices. In general:

$$a^m \div a^n = a^{m+n} \text{ or } \frac{a^n}{a^m} = a^{n-m}$$

Negative powers

Using the law for division, we know that:

$2^2 \div 2^5 = 2^{2-5} = 2^{-3}$

$2^2 \div 2^5 = \frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}$

So, $2^{-3} = \frac{1}{2^3}$ in general: $a^{-n} = \frac{1}{a^n}$

The following law helps us calculate powers raised to further powers, e.g. $(2^3)^2$.

$(2^3)^2 = (2 \times 2 \times 2)2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

So, $(2^3)^2 = 2^{3 \times 2} = 2^6$. To raise a power to a further power, multiply the indices. In general: $(a^n)^m = 2^{nxm}$

EXAMPLE: Simplify $(25x^2)^{1/2}$

$(25x^2)^{1/2} = (25)^{1/2}(x^2)^{1/2}$
$= \sqrt{25}\,(x^{2 \times 1/2})$
$= 5x^1 = 5x$

EXAMPLE: Simplify $(8x^6)^{-1/3}$

$(8x^6)^{-1/3} = (8)^{-1/3}(x^6)^{-1/3}$
$= \frac{1}{\sqrt[3]{8}}\,(x^{6 \times (-1/2)})$
$= \frac{1}{2}x^{-2} = \frac{1}{2x^2}$

Practice

i. Use the laws to find the values of these expressions:

a. $5^2 \times 5^4$ **b.** $5^{10} \div 5^2$ **c.** $10^3 \div 10$ **d.** $12^2 \times 12$ **e.** 6^0

f. $(2^5)^2$ **g.** $6^{10} \div 6^9$ **h.** $3^{-2} \div 3^2$ **i.** $5^2 \div 5^{-3}$

j. $\dfrac{2^3 \times 2^5}{(2^3)^2}$ **k.** $(3^2)^{-2}$ **l.** $\dfrac{2^3 \times 2^{-3}}{(2^2)^2}$ **m.** $\dfrac{(3^{-2})^3}{3^{-2} \times 3^{-6}}$

ii. Simplify the following

a. $x^{-2} \div \dfrac{1}{x^{-2}}$ **b.** $y(\sqrt{xy})^3$ **c.** $(9x^2)^{1/2}$ **d.** $(8x^{-3})^{1/3}$

e. $(25x^{-4})^{-1/2}$ **f.** $(81x^2)^{3/2}$ **g.** $32x^{1/2}(16x^2)^{-1/4}$ **h.** $4^{3/2}(9y^4)^{1/2}$

We can also find a law for **fractional indices**.

We know that $\sqrt{25} = 5$ and that $5 \times 5 = \sqrt{25} \times \sqrt{25} = 25$.

We can replace 25 with any number so that, $\sqrt{x} \times \sqrt{x} = x = x^1$.

Also, by the multiplication law above, we know that $x^{1/2} \times x^{1/2} = x^{1/2 + 1/2} = x^1$. So, $x^{1/2} = \sqrt{x}$

Similarly $\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = x$ and $x^{1/3} \times x^{1/3} \times x^{1/3} = x^{1/3 + 1/3 + 1/3} = x$. So, $x^{1/3} = \sqrt[3]{x}$

In general: $x^{1/n} = \sqrt[n]{x}$

EXAMPLE: Calculate a) $8^{1/3}$ b) $64^{1/3}$ c) $64^{-2/3}$

a) $8^{1/3} = \sqrt[3]{8} = 2$ because $2 \times 2 \times 2 = 8$

b) $64^{1/3} = \sqrt[3]{64} = 4$ because $4 \times 4 \times 4 = 64$

c) $64^{-2/3} = 64^{-1/3} \times 64^{-1/3} = \dfrac{1}{64^{1/3}} \times \dfrac{1}{64^{1/3}} = \dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{16}$

Practice

a. $4^{1/2}$ **b.** $4^{-1/2}$ **c.** $4^{3/2}$ **d.** $16^{1/4}$ **e.** $16^{-3/4}$ **f.** $25^{-1/2}$

g. $1000^{1/3}$ **h.** $25^{3/2}$ **i.** $16^{1/4} \times 32^{2/5}$

11.3 Equations involving indices

We can use the laws of indices to find the value of an unknown index.

EXAMPLE: Find k in the equation $x^k = \dfrac{1}{x^{-2}}$

$x^k = \dfrac{1}{x^{-2}} = (x^{-2})^{-1}$

The multiplying law $(a^n)^m = a^{nxm}$ gives

$x^k = (x^{-2})^{-1} = x^2$

So that, k = 2

EXAMPLE: Find k in the equation $x^k = \sqrt{x} \div \dfrac{1}{x^3}$

$x^k = \sqrt{x} \div \dfrac{1}{x^3} = x^{1/2} \div x^{-3}$

The division law $a^n \div a^m = a^{n-m}$ gives

$x^k = x^{1/2} \div x^{-3} = x^{1/2-(-3)} = x^{1/2+3} = x^{3/2}$

So that, k = 3/2

Practice

Find the value of k in each equation

a. $x^k = \sqrt{k}$ **b.** $y^k = 1$

c. $a^k = 1 \div a^3$ **d.** $x^k = \sqrt[3]{x^2}$

e. $x^{k+1} = (x^{-2})^{-3}$ **f.** $2^k = 16$

g. $3^k = 27$ **h.** $25^k = 5$

Glossary of Keywords

Here is a list of Mathematical words from this module. The section where the word appears is given in brackets. Find the words and what they mean - your teacher will test your memory soon!

Algebra	(1.1)	Eliminate	(5.2)
Algebraic expression	(1.1)	Substitution	(5.3)
Linear	(1.1)	Inequality	(6.1)
Degree	(1.1)	Range of values	(6.3)
Like terms	(1.1)	Region	(6.4)
General rule	(1.1)	Boundary	(6.5)
Factorise	(1.2)	Quadratic	(7.1)
Algebraic fraction	(1.3)	Identity	(7.2)
Equation	(2.1)	Difference of two squares	(7.3)
Fractional equation	(2.3)	Pythagoras' theorem	(8.1)
Graph	(3.1)	Hypotenuse	(8.1)
Coordinates	(3.1)	Quadratic equation	(9.1)
x-coordinate	(3.1)	Completing the square	(9.1)
y-coordinate	(3.1)	Quadratic formula	(9.1)
Gradient	(3.2)	Parabola	(9.6)
Formula	(4.1)	Sequence	(10.1)
Formulae	(4.1)	Pattern	(10.1)
Evaluate	(4.3)	Index form	(11.1)
Infinite	(5.1)	Index	(11.1)
Simultaneous equations	(5.1)	Indices	(11.1)
Coefficient	(5.2)	Fractional indices	(11.2)

Assessment

This assessment is written to test your understanding of the module. Review the work you have done before taking the test. Good luck!

Part 1 - Vocabulary

These questions test your knowledge of the keywords from this module. Complete the gaps in each sentence by using the words in the box. Be careful, there are 15 terms but only 10 questions!

linear	gradient	coefficient	region	hypotenuse	sequence
index	parabola	general rule	coordinates	factorise	
degree	identity	equation	formula		

a. The longest side of a right-angled triangle is the __________________.

b. The _____________ tells us the highest power of an expression.

c. A point on a graph is defined by it's _________________.

d. The _________________ tells us the slope of a line.

e. The graph of a quadratic equation is called a ______________________.

f. We use brackets when we __________________ an expression.

g. A ______________ expression has degree 1.

h. The number that is attached to an unknown in an equation is called a __________________.

i. A ____________________ is true for all numbers

j. A _________________ is a set of numbers arranged in order.

Part 2 - Mathematics

These questions test your understanding of the Mathematics in this module. Try to answer all the questions. Write your calculations and answers on separate paper.

1. Simplify these expressions

a. $2m - 3n + 3(m + n)$ b. $4(2a + b) + 2(3a - b)$ c. $2(p - q) + 2(p - q)$

2. Factorise these expressions

a. $4p - 6$ b. $6a + 4b$ c. $2ab + 2ac$

3. Solve these equations

a. $9 - 2x = 3 - 4x$ b. $4 - 2t = 2(8 + t)$ c. $3(4x - 5) - 5(2x - 3) = 4(x - 1)$

4. Solve these equations

a. $\frac{p - 4}{8} = 7$ b. $\frac{n + 3}{3} = \frac{n}{5} + 3$ c. $3(2p - 10) = \frac{4p - 7}{2}$

5. Draw the graphs of these equations

a. $y = 3x + 2$ b. $y = 3 - x$

6. Evaluate the formulae

a. $p = 2(l + w)$, when $l = 8.3$ and $w = 6.8$

b. $y = 5x + 7$, when $y = 11$

c. $s = ut + at^2/2$, when $s = 121$, $t = 5$ and $a = 8$

7. Solve the simultaneous equations

a. $3x + 2y = 14$
$5x - 2y = 18$

b. $x - 6y = 16$
$2x + 6y = 5$

c. $8x + 3y = 35$
$2x - 5y = 3$

8. Solve the inequalities

a. $7x + 2 < 5x - 4$ b. $8 - 3j < 2j + 13$ c. $2(3x - 5) > 8 - 3x$

9. Use the FOIL rule to expand the brackets

a. $(2x + 5y)(3x + 4y)$ b. $(4p + 2)(3p + 2)$ c. $(2x + 1)^2$

10. Factorise the expressions

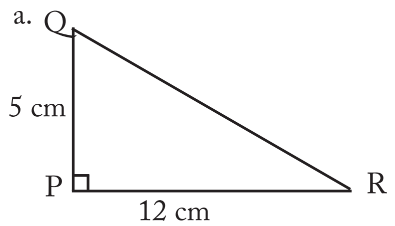
a. $x^2 - 5x + 4$ b. $5x + 6 + x^2$ c. $6a^2 + 10a + 4$

11. Simplify the expressions

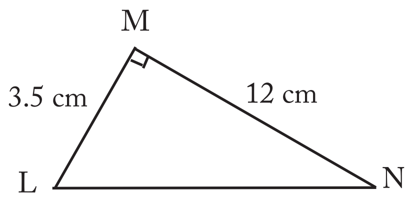
a. $\frac{x^2 - 5x + 4}{x^2 - 16}$ b. $\frac{1}{x - 1} - \frac{x - 2}{x^2 + 3x + 2}$ c. $\frac{x^2 - 5x + 4}{x^2 - 16} - \frac{x^2 + 11x + 18}{x^2 + 6x + 8}$

12. Find the length of the missing sides

a.



b.



13. Solve these equations by factorising

a. $y^2 - 3y - 4 = 0$ b. $2y^2 + 5y - 3 = 0$ c. $6y^2 - 11y - 7 = 0$

14. Solve these equations using the formula. Leave your answers in surd form

a. $x^2 - 4x + 1$ b. $x^2 - 5x + 1$ c. $4x^2 + 9x + 1$

15. Draw the graphs of these equations

a. $y = 2x$ b. $y = x^2 + 2$

16. Find the first five terms of each sequence

a. $u_n = n + 2$ b. $u^n = 3n^2 - 1$ c. $u_n = (n + 1)^2$

17. Find the formula for the n th term

a. 7, 11, 15, 19, 23,.... b. 12, 10, 8, 6, 4,..... c. 7, 4, 1, -2, -5,.....

18. Simplify the expressions

a. $2a^2b \times 3b^2c^3 \times 3c^2$ b. $7(3x^4)^2$ c. $18x^4y^3 \div 3x^3$