MATHS MODULE 1

Numbers

Student's book

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The Curriculum Project

Maths Module 1: Numbers

Contents

1. Whole Numbers

1.1 Introduction

A **whole number** is a number with no fraction or decimal part, 100, 250 and 1000 are whole numbers.

Think

When do we use whole numbers in our everyday life? Think of some examples.

Dave is very bad at Maths. One day he went to the market and spent 230 kyat on vegetables. He paid with a 1000 kyat note. The shopkeeper gave him 570 kyat change. Dave took the money and went home.

He made a mistake. What was it?

He would not have made this mistake if he was good at maths.

1.2 Place Value

The position of a number tells us its **value**.

ten thousands	thousands	hundreds	tens	units
4	**7**	**2**	**8**	**6**

Think

What is the value of the 8 in this number?
What about the 4?
Write the number in words.

Practice

i. Write these numbers in words

a) 204 **b)** 1023 **c)** 9552
d) 10,256 **e)** 81,505 **f)** 370,000

ii. Write these numbers in figures

a) seven hundred and five
b) two thousand six hundred and fifty two
c) twenty two thousand five hundred
d) one million two hundred and fifty seven thousand

iii. Make as many different numbers as you can from the figures 2, 6, 7, 0, 3. Arrange the numbers in **order**, starting with the smallest.

1.3 Rounding

We don't always need to know a number exactly. Sometimes we use **rounding** to give an **estimate**.

Example - The population of Ming Town is 48,492.
To round to the nearest *hundred* we look at the number in the *tens* column.
It is greater than 5 so we round up. To the nearest hundred the population of Ming Town is 48,500.

0 1 2 3 4 | 5 6 7 8 9

If the number is **less than** 5, we **round down**.

If the number is **greater than or equal to** 5, we **round up**.

Practice

a) What is the population of Ming Town to the nearest thousand. Did you round up or down? Why?

b) The exact number of tickets sold for an Ironcross concert in Rangoon was 9782. How many tickets were sold to the nearest hundred?

c) Yesterday the Yangon Times newspaper sold 56,792 copies. How many copies were sold to the nearest thousand?

d) The population of Myanmar is 47,382,683.
To the nearest million how many people live in Myanmar?

We can also use rounding to check our calculations.

Example - Use rounding to check that the answer to the sum is correct.

$$\begin{array}{r} 2389 \\ +\ 1467 \\ \hline 3856 \\ \hline \end{array}$$

2389 to the nearest hundred is 2400, 1467 to the nearest hundred is 1500. 2400 + 1500 = 3900.

This estimate is close to our answer, so it is probably correct.

If the answer and the estimate are very different, then we know something is wrong!

1.4 Addition and Subtraction

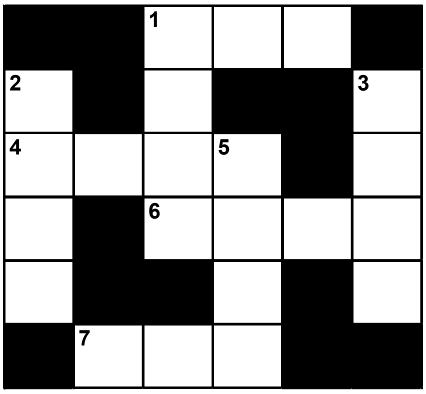
One important **property** of **addition** is that we can add numbers in any order and the answer will be the same. 5 + 3 + 8 = 8 + 5 + 3. This property is called the **commutative law** of addition.

The commutative law is not true for **subtraction**. $5 - 3 - 8 \neq 8 - 5 - 3$.

Addition is commutative. Subtraction is not commutative.

Practice

i. Solve the crossword using the clues. Use rounding to check your answers.



Across

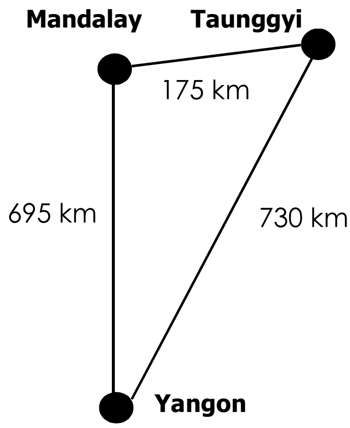
1. Add: 165 + 256
4. Add: 943 + 468
6. Add: 1159 + 4457
7. Add: 439 + 320

Down

1. Subtract: 5270 - 655
2. Subtract: 2563 - 417
3. Subtract: 2748 - 1584
5. Subtract: 2252 - 553

ii. Min Too has to go from Yangon to Taunggyi and then to Mandalay. How far is his journey?

He goes back to Yangon directly. How much shorter is his journey home?



iii. Ben Nevis is the highest mountain in Britain. It is 1343 m high. Mount Everest is the highest mountain in the world. It is 8843 m high. How much higher is Mount Everest than Ben Nevis?

Village population figures - June 2009	Female	Male	Total
District 1			
Truro	9204	9878	19082
Morecambe	1740	1894	3634
Bognor	7613	8307	
Brighton	7384	7650	15034
Total populations for District 1	25941	27729	53670
District 2			
Scarborough	23732	24760	48492
Hull	9439	10172	19611
Blackpool	6529	6936	13465
Total populations for District 2		41868	81568
District 3			
Southend	314	286	
Cleethorpes	2251	2198	4449
Skegness	5198	5373	10571
Total populations for District 3			
Total populations for all villages		77454	150858

iv. The table shows the number of people who live in different villages in Oompaland.

Answer the questions to complete the table. Before you fill in the gaps use rounding to check your answer.

a) What is the total population of Southend village?

b) What about the population of Bognor?

c) What is the total number of females in the villages in District 2?

d) Use your answer for **c)** to calculate the total number of females in all the villages. How else could you calculate this figure?

1.5 Multiplication and Division

Think

Solve each calculcation. Now write the numbers in a different order and solve again.
What do you notice?

a) 2 x 4 x 6 =

b)10 ÷ 2 ÷ 5 =

c) 6 x 9 x 8 =

d) 32 ÷ 4 ÷ 2 =

The exercise shows us that **Muliplication** is commutative. The order you calculate is not important. **Division** is not commutative. The order we calculate is important.

The commutative law can make calculations easier:

Example - Calculate 10 x 3 x 22.
10 x 3 x 22 = 22 x 3 x 10 = 66 x 10 = 660

Think

When we multiply by 10 we add a zero to the number.
What about when we multiply by 100? 1000?

Here are two more methods to make multiplication easier:

Example - Splitting method.
Calculate 17 x 6. We know 6 = 2 x 3.
17 x 6 = 17 x 2 x 3 = 34 x 3 = 112

Example - Rounding method. Calculate 49 x 6.
First, we round 49 to 50.
49 x 6 = 50 x 6 - 1 x 6 = 300 - 6 = 294

Practice

i. Choose a method for each question and calculate:

a) 47 x 7
b) 10 x 5 x 11
c) 13 x 9
d) 21 x 8
e) 10 x 35 x 2
f) 67 x 8
g) 8 x 21

ii. Solve the following:

a) A school day is 7 hours.
How many minutes is this?

b) A gallon of petrol costs 280 kyat.
How much do 12 gallons cost?

c) Mama noodles cost 60 kyat a packet.
How much do 52 packets cost?

For larger numbers we use **long multiplication**

> **Example -** Find 84 x 26
>
> 84 x 26 = 84 x 6 + 84 x 20. So,
>
> ```
>      8 4
>    x 2 6
>   -------
>    5 0 4     (84 x 6)
> + 1 6 8 0    (84 x 20)
>   -------
>  2 1 8 4     (504 + 1680)
> ```

Practice

Use long multiplication to calculate

a) 355 x 25
b) 462 x 33
c) 589 x 62
d) 241 x 32
e) 1251 x 28
f) 146 x 259

We can use similar methods for division as we do for multiplication.

Think

Which multiplication method can be used to calculate 816 ÷ 6? What is the answer?

Practice

Use the splitting method to calculate

a) 56 ÷ 4
b) 90 ÷ 15
c) 112 ÷ 16
d) 720 ÷ 24

> **Example -** Use **long division** to calculate $21\overline{)2678}$.
>
> $21\overline{)2678}$
>
> | | |
> |---|---|
> | 21 | There is 1 twenty-one in 26, **remainder** 5. |
> | 57 | |
> | 42 | There are 2 twenty-ones in 57, r 15. |
> | 158 | |
> | 147 | There are 7 twenty-ones in 26, r 11. |
> | 11 | |
>
> $21\overline{)2678}$ = 127 r 11

Practice

Use Long division to calculate:

a) $48\overline{)576}$
b) $35\overline{)8050}$
c) $18\overline{)4824}$
d) $38\overline{)9728}$
e) $33\overline{)8392}$
f) $27\overline{)6732}$

> ## Activity
>
> Multiply the number 123456789 by 3, then multiply the result by 9.
>
> Now multiply 123456789 by 5 and then again by 9. What do you notice?
>
> What will the answer be if you multiply by 4 then 9? What about multiplying by 7 and then 9?

1.6 Order of Operations

In maths an **operation** is something we do to a number. Multiplication, division, addition and subtraction are all operations. If we have a calculation with more than one operation, the order we operate is important.

Think

a) Look at these calculations.
What order should we do the operations?

6 + 4 x 7 -13
37 - 35 ÷ 5
8 x 4 + 15 ÷ 3

b) Complete the sentence:

If we have more than one operation, we do the ___________ and ___________ first. Then we do the ___________ and ___________.

Practice

Complete the calculations:

a) $5 \times 4 + 2 \times 3$
b) $16 + 3 \times 4 - 2$
c) $9 - 8 + 5 \times 2$
d) $8 + 4 \times 0$
e) $5 \times 4 \div 10 + 6$
f) $19 + 3 \times 2 - 8 \div 2$
g) $4 \times 2 - 6 \div 3 + 3 \times 2 \times 4$

If a calculation contains **brackets** then what is inside the brackets must be calculated *first*.

> **Example** - Calculate $2 \times (3 + 5)$
> $2 \times (3 + 5) = 2 \times 8 = 16$

3^2 is read as **three squared** or **to the power of 2**.
$3^2 = 3 \times 3 = 9$.
3^3 is read as **3 cubed** or **3 to the power of 3**.
$3^3 = 3 \times 3 \times 3 = 27$.

If a calculation contains **powers** then we calculate them *after* the brackets.

> **Example** - Calculate $(3 + 5)^2 - 8 \times 4$
> $(3 + 5)^2 - 8 \times 4 = 8^2 - 8 \times 4 = 64 - 32 = 32$

> **Example** - Calculate $4+\frac{8}{5-3}$
> $4+\frac{8}{5-3} = 4+\frac{8}{2} = 4+4 = 8$

$\frac{8}{5-3}$ means $8 \div (5-3) = 8 \div 2 = 4$

Practice

i. Insert the missing operation to make each statment correct:

a) 5 - 4 + 1 = 5 __ 1 __ 4
b) 120 ÷ 6 x 2 ÷ 10 = 120 __ 10 __ 6 __ 2
c) 19 + 7 - 12 + 5 = 19 __ 12 __ 5 __ 7
d) 18 ÷ 3 + 2 x 6 - 4 = 2 __ 6 __ 4 __ 18 __ 3

ii. Put brackets in the statements to make them true:

a) $12 \div 3 \times 2 = 2$
b) $4 \times 5 + 7 = 48$
c) $8 + 6 \div 3 + 2 = 12$
d) $27 - 5 \times 9 - 5 = 88$

iii. Calculate the value of:

a) $2 \times (7 - 2) \div (16 - 11)$
b) $12 - 8 - 3 \times (9 - 8)$
c) $(5 + 3) \times 2 + 10 \div (8 - 3)$
d) $8^2 + 5^2 + 6^2$
e) $28 - (9 - 7)^2$
f) $72 \div (5 - 2)^2$
g) $(52 - 42)^3$
h) $\frac{(2+3)^2}{(14-9)^2}$

iv. Write each calculation using brackets and solve:

a) Three students went to the shop after school. One student bought 3 cakes for 20 kyat each. One student bought 3 packets of coffee mix for 30 kyat each. The third student bought 3 sweets for 10 kyat each. How much money did they spend all together?

b) Two sisters have money boxes. There are five 20 kyat notes and four 10 kyat notes. The second box has six 20 kyat coins and ten 10 kyat notes. What is the total amount of money?

c) Hledan youth club had 82 members at the beginning of 2006. In March 32 left. In May 27 joined and 12 more left. How many students were in the club at the beginning of June?

Use this diagram to help you remember the **order of operations:**

brackets → **powers** → **division** and **multiplication** → **addition** and **subtraction**

Think

Make different answers by putting brackets into the calculation 3 x 5 - 2 x 7 +1.
For example: (3 x 5) - (2 x 7) + 1 = 2.

Write some more examples and give them to your partner. Put brackets into your partner's examples to make different answers.

Activity - For this you need ten cards numbered 0 - 9.

Turn the cards over. Take two cards to make a 2-digit number. Take four more cards.

With the four cards, try to make an expression using brackets, operations and powers that is equal to your 2-digit number.

You get points for the difference between your answer and the 2-digit number.
Try to get a low score.

Example. My two digit number is 47. My four numbers are 1, 3, 8, 5.

(8 x 5) + 1 + 3 = 44 scores 3 points because 47 - 44 = 3.
(1+ 8) x 5 + 3 = 48 scores 1 point because 48 - 47 = 1.

Try once. Shuffle the cards and take some new numbers.
Play 10 times. The person with the lowest score wins.

2. Factors and Multiples

2.1 Introduction

When 12 is divided by 3, the remainder is zero. We say that 3 is a **factor** of 12.

Think

What are the other factors of 12? What are the factors of 18?

We can write a number as the **product** of two factors: 1 x 12, 3 x 4 , 2 x 6 are all equal to 12.

Practice

Write each number as a product of two factors:

a) 18
b) 24
c) 27
d) 36
e) 64
f) 80
g) 96
h) 144

12 divided by 2 is equal to a whole number. We say that 12 is a **multiple** of 2.
12 is also a multiple of 1, 3, 4, 6 and 12.

Practice

i. Complete the sentences:

a) 15 is a multiple of ___, ____, _____ and 15
b) 27 is a multiple of 1, ___, ____ and ____
c) 40 is a multiple of ___, 4, ___, ___,___,___,___
d) 41 is a multiple of ____ and ____

ii. Answer the following:

a) Write down all the multiples of 3 between 20 and 40.
b) Write down all the multiples of 7 between 25 and 60.
c) Write down all the multiples of 13 between 25 and 70.

2.2 Prime Numbers

Some numbers only have two factors. The factors of 3 are 1 and 3. The factors of 5 are 1 and 5.

3 and 5 are called **prime numbers.** 1 is not a prime number because it only has one factor.

Think

Is this statement true or false?
Explain your answer:

'The only even prime number is 2'

Practice

Answer the following:

a) Which numbers are prime numbers?:
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

b) Write down all the prime numbers between 21 and 30.

c) Write down all the prime numbers between 30 and 50.

Activity - Look at the numbers below. 1 is crossed out. 2 is circled.

Write the numbers 1-100 in your book.
Cross out all the multiples of 2.
Circle 3 and cross out all the multiples of 3.
Circle 5 and cross out all the multiples of 5.
Circle 7 and cross out all the multiples of 7.

Continue until all numbers are circled or crossed out.

Write out all the numbers that are not circled or crossed out. What are these numbers?

~~1~~ (2) 3 4 5

2.3 Indices

We learnt in Section 1.5 that 3^3 is read as '3 cubed' or '3 to the power of 3'. $3^3 = 3 \times 3 \times 3 = 27$.
The power is called the **index.** (The plural of index is **indices**). 3^3 is 3 x 3 x 3 written in **index form.**

Practice

i. Write the numbers in index form:

a) 3 x 3
b) 2 x 2 x 2
c) 5 x 5 x 5 x 5
d) 7 x 7 x 7 x 7 x 7
e) 13 x 13 x 13 x 13
f) 2 x 2 x 3 x 3
g) 3 x 3 x 3 x 5 x 5
h) 3 x 11 x 11 x 2 x 2
i) 13 x 5 x 13 x 5 x 13

ii. Find the value of:

a) 3^3
b) 2^5
c) 5^2
d) 3^4
e) 7^2
f) $2^3 \times 3$
g) $2^3 \times 3^2$
h) $3^2 \times 5^2$
i) $2 \times 3^3 \times 7$

Remember that multiplication is commutative!

2.4 Prime Factors

A **prime factor** is a prime number that is a factor of another number. 3 is a prime number and a factor of 9 so it is a prime factor of 9.

These rules can help us find prime factors:

- A number is divisible by 2 if the last digit is even.
- 3 if the sum of the digits is divisible by 3.
- 5 if the last digit is 0 or 5.

Practice

Are these numbers divisible by 2, 3 or 5?:

a) 525 **b)** 747 **c)** 740 **d)** 1424

Any whole number that is greater than 1 can be written as a product of prime factors.

Example - Write 720 as a product of prime factors.

Method 1: Start by dividing by 2.

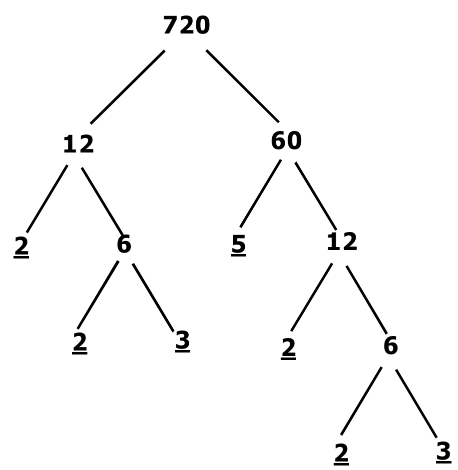
2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

45 is not divisible by 2.
We try to divide by 3.

5 is not divisible by 3.
We divide by 5.

We have 720 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^4 \times 3^2 \times 5$

Method 2: Use a factor tree.



The underlined numbers are the prime factors.

We have 720 $= 2 \times 2 \times 3 \times 5 \times 2 \times 2 \times 3$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^4 \times 3^2 \times 5$

Think

To use the factor tree we divided 720 into 2 non-prime factors, 12 and 60.
Find 2 different non-prime factors of 720. Use them in a factor tree to find the prime factors of 720.

2.5 Highest Common Factor (H.C.F.)

The **Highest Common Factor (H.C.F)** of two or more numbers is the biggest number that is a factor of both numbers.

Example - Find the H.C.F of 16 and 24.

The factors of 16 are 1, 2, 4, 8 and 16.
The factors of 24 are 1, 2, 4, 6, 8, 12 and 24.

1, 2, 4 and 8 are factors of 16 *and* 24. 8 is the biggest factor. 8 is the Highest Common Factor.

Practice

Find the Highest Common Factor of:

a) 9, 12
b) 12, 24
c) 14, 42
d) 30, 45, 90
e) 39, 13, 26
f) 36, 44, 52

2.6 Lowest Common Multiple (L.C.M.)

The **Lowest Common Multiple (L.C.M.)** of two numbers is the smallest number that is a multiple of both numbers.

Example - Find the L.C.M. of 6 and 8.

The multiples of 6 are: 6 12 18 **24** 30 36 42 **48** 54 60 66 **72**.....

The multiples of 8 are: 8 16 **24** 32 40 **48** 56 64 **72** 80 88 96......

We can see that common multiples of 6 and 8 are: 24, 48 and 72. 24 is the smallest multiple. 24 is the Lowest Common Multiple.

Practice

Find the Lowest Common Multiple of:

a) 3 , 5
b) 6, 8
c) 5, 15
d) 12, 16
e) 4, 5, 6
f) 18, 24, 36

Activity - For this you need twenty folded pieces of paper numbered 1-20.

Your teacher will divide the class into groups of 4 and then into two teams of two people.

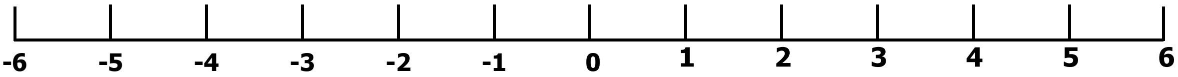
The first team picks two numbers. The second pair has to find the lowest common multiple of the two numbers. If they are right, they get one point. Fold the numbers and put them back.

Now the second pair picks two numbers and the first pair has to find the lowest common multiple.
The first team to score 10 points is the winner.

3. Negative Numbers

3.1 Introduction

Look at the number line. Numbers to the right of zero are **positive numbers**. They are *greater than* zero. The numbers to the left of zero are **negative numbers.** They are *less than* zero.



The postitive number 5 means +5, but we don't write the '+' sign. When we write negative numbers, we put a '-' sign in front of the number.

Think

We know that 5 > 3 (5 is greater than 3) and that 2 < 4 (2 is less than 4). What about negative numbers? Put the correct sign between the numbers below.

a) -2 -4 **b)** -3 -1 **c)** -4 1 **d)** -4 -2 0
e) 5 1 -3 **f)** -3 -9 -12

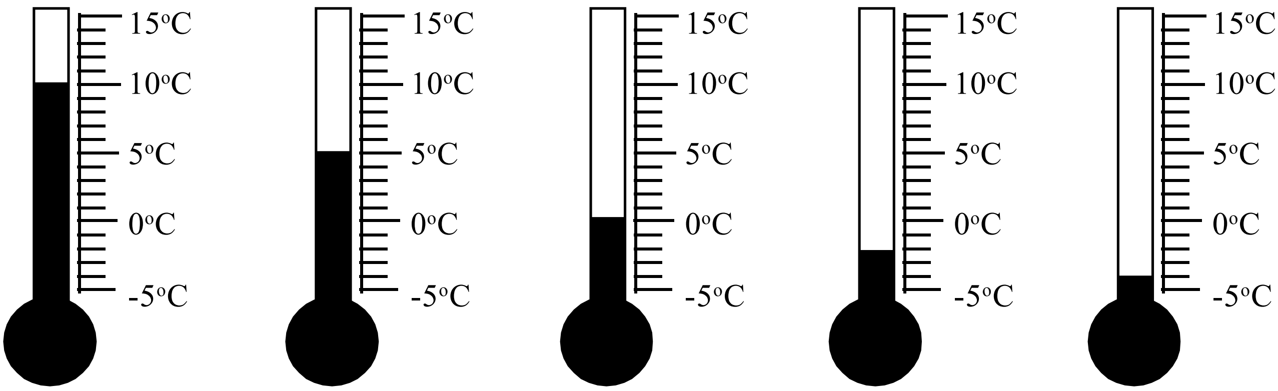
We see that as a negative number increases, its value decreases.

3.2 Temperatures

Temperature is measured in degrees Celsius or degrees centigrade. Eight degrees Celsius is written as 8°C. In Science you learn that water freezes at 0°C. The temperature where you live is always greater than 0°C. In England, America and many other countries the temperature is sometimes less than 0°C. Temperatures less than 0°C are written as negative numbers, such as -5°C. Temperatures are measured using thermometers.

Practice

Write the temperature given by each thermometer. The first one is done for you.



10°C

Now look at the map of Britain and answer the questions.

a) Which city has the warmest temperature?
b) How many degrees below freezing is Glasgow?
c) How many degrees above freezing is Swansea?
d) Is Aberdeen colder than Glasgow?
e) Which city has the coldest temperature?

3.3 Addition and Subtraction

We use these rules when we add and subtract with negative numbers:

+ (+2) = 2	- (+2) = -2
+ (-2) = -2	- (-2) = 2

Example - Find 5 + (-7)
5 + (-7) = 5 - 7 = -2

Example - Find 3 - (-2) + (-8) + 1
3 - (-2) + (-8) + 1 = 3 + 2 - 8 + 1 = -2

Practice

i. Find:

a) 3 + (-6)
b) -2 - (-3)
c) -5 - (+7)
d) -(+1) - (+5)
e) -(+3) + (+5) - (+5)
f) 12 + (+8) - (-4)
g) 2 - (-4) + 6
h) -2 - (+12)
i) - 9 + (-2) - (-3)

ii. Find the following. Remember, brackets first!

a) 3 - (4 - 3)
b) 6 + (8 - 15)
c) 5 + (7 - 9)
d) 5 - (6 - 10)
e) (4 - 8) - (10 - 15)
f) (3 - 8) - (9 - 4)

iii. The numbers in this triangle are found by adding together the two numbers above. Complete it.

5	-4	2	-1	-3		0	5
1	-2			-11			
-1							

3.4 Multiplication and Division

We already know that 4 x 2 = 8 and that 4 ÷ 2 = 2. For negative numbers we use these rules:

If one number is negative, the answer is negative:

(-3) x 2 = -6 and 3 x (-2) = -6

4 ÷ (-2) = -2 and (-4) ÷ 2 = -2

If both numbers are negative, the answer is positive:

(- 3) x (-2) = 6

(-4) ÷ (-2) = 2.

Practice

Find:

a) 6 x (-4) **b)** 7 x (-2) **c)** 8 x (-2) **d)** (-8) x (-3) **e)** (-12) x (-11) **f)** (-16) x (-7)
g) (-6) ÷ 2 **h)** (-15) ÷ 3 **i)** (-28) ÷ 7 **j)** (-28) ÷ (-4) **k)** (-72) ÷ (-9) **l)** (-144) ÷ (-12)

Why does a negative number multiplied by a negative number give a postive answer?

First, we use some things we already know: 3 + (-3) = 0 and 2 x (0) = 0.

These tell us that: 2 x (0) = 2 x (3 + (-3)) = 0.

We can **expand** the brackets: 2 x (3 + (-3)) = 2 x 3 + 2 x (-3).

So: 2 x (0) = 2 x 3 + 2 x (-3) = 6 + 2 x (-3) = 0.

For this to be true, 2 x (-3) must be equal to -6 so: 2 x (0) = 6 + (-6) = 6 - 6 =0.

From this example we learn that a negative multiplied by a postive gives a negative answer.

However, it is also true that -2 x (0) = 0 and that -2 x (0) = -2 x (3 + (-3)) = -2 x 3 + (-2) x (-3) = 0.

We already know that -2 x 3 = -6. So, -2 x (0) = -6 + (-2) x (-3) = 0.

This is only true if (-2) x (-3) is equal to 6 so that -2 x (0) = -6 + 6 = 0.

From this example we learn that a negative multiplied by a negative gives a positive answer.

4. Decimals

4.1 Introduction

The **decimal point** separates the whole numbers from the parts of the whole.

Think

units		tenths	hundredths	thousandths
8	•	4	7	5

The number in the **tenths** column is 4. There are 4 tenths. In decimal notation this is 0.4

How many **hundreths** are there? **Thousandths**?
Can you write these as decimals?
Do you know how to say this number?

Practice

i. Practice saying these numbers

a) 0.8725 **c)** 72.156874
b) 24.9182 **d)** 0.05146

ii. Write the underlined digits in words.

iii. Write these as decimal numbers

a) fifteen, three tenths and four hundreths
b) eleven and seven hundreths
c) twenty seven and thirty five thousandths
d) five tenths and six thousandths

4.2 Addition of Decimals

We add decimals in the same way we add whole numbers.

Example - Find 5.3 + 6.8

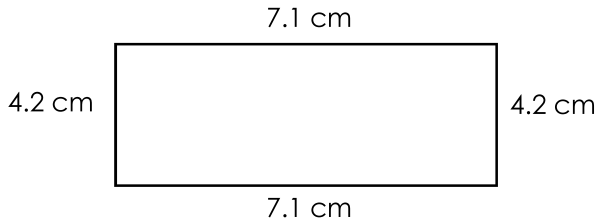
$$\begin{array}{r} 5.3 \\ +\ 6.8 \\ \hline 12.1 \end{array}$$

Practice

i. Solve the following

a) 7.2 + 3.6
b) 0.013 + 0.026
c) 3.87 + 0.11
d) 0.0043 + 0.263
e) 4.62 + 0.078
f) 0.32 + 0.032 + 0.0032
g) 7.34 + 6 + 14.034

ii. Find the **perimeter** of the **rectangle**
(perimeter is the distance all the way around):



Activity - Copy the grid of numbers into your exercise book

4.33	0.59	2.36	5.608	3.182	0.57	0.649
6.25	1.89	5.81	3.218	1.14	2.98	3.902
3.722	0.9	3.7	5.959	6.27	6.804	0.098
0.13	5.91	3.241	0.68	1.291	2.99	4.2

Cross out pairs of numbers that add up to between 5 and 7 (for example 4.33+1.89= 6.22).
You have 2 minutes to cross out as many pairs as possible.
Your score is the sum of all the remaining numbers.
The person with the lowest score is the winner.
Try again with your partner to reduce your score. What is the lowest possible score?

4.3 Subtraction of Decimals

We also subtract decimals in the same way we subtract whole numbers.

Example - Find 24.2 - 13.7

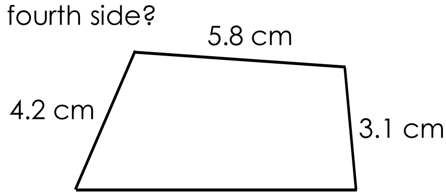
$$\begin{array}{r} 24.2 \\ -\ 13.8 \\ \hline 10.5 \end{array}$$

Practice

i. Solve the following:

a) 9.6 - 1.8
b) 17.23 - 0.36
c) 7.063 - 0.124
d) 3.2 - 0.26
e) 11 - 8.6
f) 0.73 - 0.0006
g) 9.2 + 13.21 - 14.6

ii. The perimeter of the shape is19 cm.
What is the length of the fourth side?



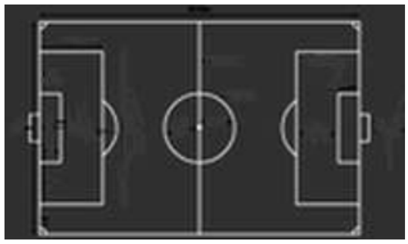
4.4 Changing Units - Length

The **metric units** of **length** are **kilometre (km), metre (m), centimetre (cm)** and **millimetre (mm).**

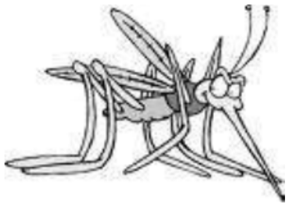
Think

Which unit of length is used to measure the following:

a) The distance between Yangon and Mandalay?

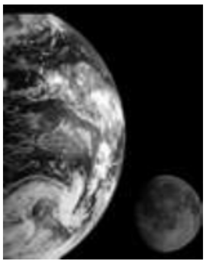


b) The length of a football pitch?



c) The length of a mosquito?

d) The width of your classroom?



The distance to the moon is measured in kilometres.
Can you guess how far it is?

To compare lengths we write them with the same units. We use the following relationships:

1 km = 1000 m	1 m = 100 cm
1 m = 1000 mm	1 cm = 10 mm

To change from large units to small units we use multiplication.
To change from small units to large units we use division.

Example - Write 3.5 m in centimetres.
Use 1 m = 100 cm
3.5 m = 3.5 x 100 cm = 350 cm.

Example - Write 580 m in kilometres.
Use 1 km = 1000 m
580 m = 580 ÷ 1000 km = 0.58 km.

Practice

i. Write the quantity in the smaller units given in brackets:

a) 2 m (cm)
b) 3 cm (mm)
c) 1.5 m (cm)
d) 9.2 m (mm)
e) 2 km (mm)

ii. Write the quantity in the larger units given in brackets:

a) 300 mm (cm)
b) 150 cm (m)
c) 12 cm (m)
d) 1250 mm (m)
e) 2850 m (km)

4.5 Changing Units - Mass

The **metric units** of **mass** are **tonne (t), kilogram (kg) and gram (g) and milligram (mg).**
We use the following relationships:

1 t = 1000 kg	1 kg = 1000 g	1 g = 1000 mg

We use grams and kilograms for things we use everyday. We use tonnes for heavier things.

Think

Which unit of mass is used to weigh:

a) Your friend?

c) A jar of coffee?

b) A truck?

d) 100 elephants?

We change units in the same way as before.

Example - Write 2.35 t in grams.
2.35 t = 2.35 x 1000 kg = 2350 kg
2350 x 1000 g = 2,350,000 g

First we change tonnes to kilograms.
Then we change kilograms to grams.

Practice

i. Write the quantity in the smaller units given in brackets:

a) 12 t (kg)
b) 13 kg (g)
c) 0.3 g (mg)
d) 0.7 t (g)
e) 0.72 kg (mg)

ii. Write the quantity in the larger units given in brackets:

a) 1500 g (t)
b) 1500 mg (g)
c) 5020 g (kg)
d) 190 kg (t)
e) 86 g (kg)

4.6 Adding and Subtracting Quantities

To add or subtract two or more quantities they must have the same units.

Think

How do we change to kilograms?

Example - Write 1 kg + 158 g in grams.
1 kg = 1000 g.
1 kg + 158 g = 1000 g + 158 g = 1158 g.

Practice

i. Solve the following giving your answer in the unit given in brackets:

a) 5 m + 86 cm (m)
b) 36 cm - 87 mm (m)
c) 116 g + 0.93 kg (g)
d) 52 mg + 87 g (kg)
e) 3.9 m + 582 mm (cm)
f) 20 g - 150 mg (g)

ii. Solve the following:

a) Aung Aung goes to the market and buys 2 kg of potatoes, 250 g of coffee and 600 g of beans. What is the total weight in kilograms?

b) To visit his friend in Dagon Min Thaw walks 900m to the road and then travels 32.65 km by linecar. His friend's house is 450 m from the bus station. How far does he travel in kilometres?

4.7 Multiplying Decimals by Whole Numbers

We multiply a decimal number and a whole number in the same way as we multiply two whole numbers.

Example - Find 2.68 x 31

  2 . 6 8
x   3 6
$1\ 6_4.0_4 8$
$8_2 0_2.4\ 0$ — Don't forget to add the zero.
9 6 .4 8

Think

How do we order the quantities, 45 cm, 0.4 m, 360 mm and 0.002 km? What do we do first?

Practice

Calculate the following:

a) 812.9 x 4
b) 0.126 x 4
c) 9 x 1.43
d) 53.72 x 6
e) 812.9 x 43
f) 0.126 x 47
g) 92 x 1.43
h) 53.72 x 64

i) Some students have to build a new wall for the classroom using bamboo. They need 23 pieces of bamboo that are 3.67 m long. What is the total length of bamboo needed?

4.8 Dividing Decimals by Whole Numbers

We also a divide decimal number and a whole number in the same way as we divide two whole numbers.

Example - Find $0.45 \div 5$

$$5\overline{)0.45} = 0.09 \quad (5 \times 9 = 45)$$

5 does not go into 4 so we write a zero in the tenths column.

Example - Find $4.2 \div 25$

```
     0.168
25 ) 4.200
     25
     170
     150
      200
      200
```

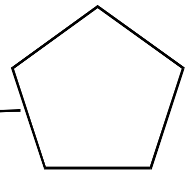
Practice

i. Calculate the following:

a) $0.672 \div 3$ **b)** $26.6 \div 7$ **c)** $0.6552 \div 6$ **d)** $0.0285 \div 5$
e) $9.45 \div 21$ **f)** $0.864 \div 24$ **g)** $71.76 \div 23$ **h)** $0.2585 \div 25$

ii. A **pentagon** is a 5-sided shape. The perimeter of the pentagon shown is 16.24 cm. What is the length of one side?

All the sides are equal



4.9 Multiplying Decimals

Think

Earlier we learnt that $0.2 = 2 \div 10$. How can use this to calculate 0.3×0.2?

Practice

Calculate the following:

a) 0.04 x 0.2 **b)** 0.003 x 0.1 **c)** 0.3 x 0.002 **d)** 0.7 x 0.001
e) 0.07 x 0.008 **f)** 8 x 0.6 **g)** 0.2 x 0.008 **h)** 4 x 0.0009

Compare the number of the **decimal places** in the answer with the those in the numbers being multiplied. What do you notice?

Example - Find 0.08 x 0.4

8 x 4 =32

0.08	x	0.4	=	0.032
(2 places)		(1 place)		(3 places)

Example - Find 0.252 x 0.4

```
  252
x   4
 1008
```

0.252	x	0.4	=	0.1008
(3 places)		(1 place)		(4 places)

Practice

Calculate the following:

a) 0.751 x 0.2 **b)** 5.6 x 0.02 **c)** 0.16 x 0.005 **d)** 0.5 x 0.005
e) 310 x 0.04 **f)** 0.68 x 0.543 **g)** 1.36 x 0.082 **h)** 0.0072 x 0.034

4.10 Division by Decimals

Example - Calculate $0.012 \div 0.06$

$0.012 \div 0.06$ can be written as $\frac{0.012}{0.06}$

$$\frac{0.012}{0.06} = \frac{0.012 \times 100}{0.06 \times 100}$$ — Multiplying top and bottom by the same number does not change the value.

$$= \frac{1.2}{6}$$ — It is easier to divide by a whole number.

$$= 0.2$$ — This is the answer!

Practice

Calculate the following:

a) $0.48 \div 0.04$ **b)** $3.6 \div 0.06$ **c)** $0.84 \div 0.07$ **d)** $0.168 \div 0.014$

e) $20.8 \div 0.0004$ **f)** $0.00132 \div 0.11$ **g)** $4.96 \div 1.6$ **h)** $0.0204 \div 0.017$

4.11 Decimal Places

We already know how to round whole numbers. We round to a given number of **decimal places (d.p.)** in a similar way.

Think

Look at the number on the right. How do we round it to 1 d.p.? What about 2 d.p?

4 3 . 0 5 7

1st 2nd 3rd

decimal place

Practice

i. Round the following to the nearest whole number:

a) 13. 9 **b)** 109.7 **c)** 152.4 **d)** 0.98

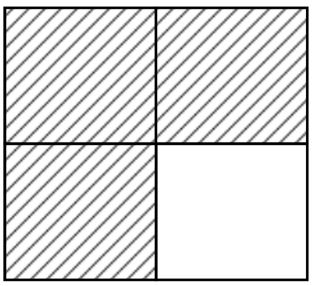
ii. Round the following to the number of decimal places given in brackets:

a) 1.2671 (2 d.p.) **b)** 0.0416 (3 d.p.) **c)** 3.9949 (2 d.p.) **d)** 8.0293 (2 d.p.)

iii. Look at the division questions i. in Section 4.8. Round your answers to 2 d.p.

5. Fractions

5.1 Introduction



Think

How many parts is the square divided into?
How many parts are shaded?
Do you know how to write this as a fraction?

Numerator

——

Denominator

The bottom number in a fraction is the total number of parts. It is called the **denominator**.

The top number is the number of parts we are counting. It is called the **numerator**.

The fraction above is called a **proper fraction** because the numerator is *less than* the denominator.

Practice

i. There are 60 minutes in one hour. What fraction of an hour is:

a) 1 minute? **b)** 7 minutes? **c)** 35 minutes? **d)** 55 minutes?

ii. You go to school 5 days every week. What fraction of the week are you at school?

iii. To travel from Mandalay to Mawlamyine I take a train to Rangoon which costs \$35, then I take a bus to Moulmein for \$9. What fraction of the total cost is the bus to Mawlamyine?

iv. Which subject do you like most, Maths, English or Social Studies? Complete the table after the class has voted.

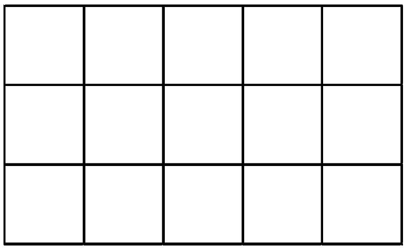
Maths	English	Social Studies	Total

What fraction of the class like Maths the most? What about English and Social Studies?

v. Myint San prepared an area for a vegtable garden. Using rope she divided the area into 15 equal squares. What fraction of the whole garden is given by one square?

She planted corn in 3 squares and green beans in 2 squares. Shade these squares on the diagram.

What fraction of the whole garden has been planted?



5.2 Improper Fractions and Mixed Numbers

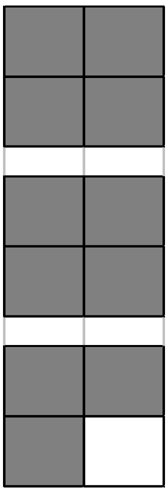
If the numerator is greater than the denominator then the fraction is an **improper fraction**.

In the diagram one square is made of 4 parts. There are 11 shaded parts in total, so we write: $\frac{11}{4}$

An improper fraction can also be written as a **mixed number** - part whole number and part fraction. In the diagram there are 2 whole squares and $\frac{3}{4}$ of another square shaded, so we write: $2\frac{3}{4}$

$$\frac{11}{4}$$

improper fraction



$$2\frac{3}{4}$$

mixed number

Think

What do we learn about $\frac{11}{4}$ and $2\frac{3}{4}$ from this example?

> **Activity -** Write some examples of proper fractions, improper fractions and mixed numbers below. Show them to your partner and ask him/her to classify them.

We can write mixed numbers as improper fractions and improper fractions as mixed numbers.

> **Example** - Write $3\frac{5}{8}$ as an improper fraction.
>
> $3 = \frac{3 \times 8}{8} = \frac{24}{8}$. $3\frac{5}{8} = \frac{24}{8} + \frac{5}{8} = \frac{29}{8}$.

> **Example** - Write $\frac{32}{5}$ as a mixed number.
>
> Divide 32 by 5: $5\overline{)32}$ = 6 remainder 2.
>
> In words, this is 6 wholes and 2 parts. So, $\frac{32}{5} = 6\frac{2}{5}$.

Practice

Write the improper fractions as mixed numbers:

i. a) $\frac{11}{5}$ b) $\frac{12}{7}$ c) $\frac{27}{8}$ d) $\frac{84}{9}$ e) $\frac{91}{6}$ f) $\frac{121}{9}$

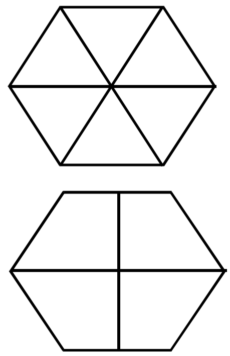
Write the mixed numbers as improper fractions:

ii. a) $2\frac{3}{4}$ b) $4\frac{1}{6}$ c) $3\frac{11}{15}$ d) $7\frac{5}{12}$ e) $4\frac{7}{9}$ f) $10\frac{3}{7}$

5.3 Equivalent Fractions

Think

Shade 3 parts of the first shape and 2 parts of the second.
What do you see?
Look at the shapes and complete the statement: ____ = ____



The fractions above are **equivalent**. They have the same value.
Equivalent sets of fractions can be found using a **multiplication wall**.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60

Think

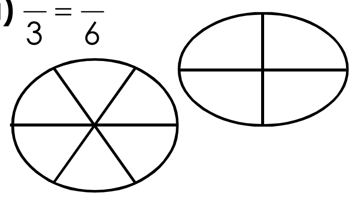
Look at the 2nd and 5th rows of the multiplication wall. Complete the set of equivalent fractions:

$\frac{2}{5} = \frac{4}{10} =$

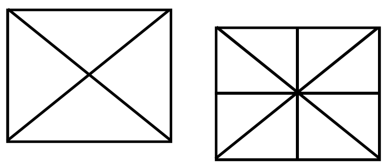
Practice

Shade the shapes to show that the fractions are equivalent

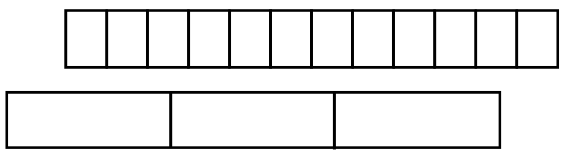
a) $\frac{1}{3} = \frac{2}{6}$



b) $\frac{2}{4} = \frac{4}{8}$



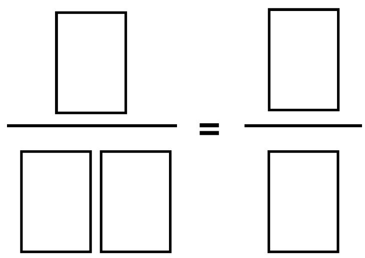
c) $\frac{2}{3} = \frac{8}{12}$



Use the multiplication wall to make sets of equivalent fractions for **a)**, **b)** and **c)**.

Activity - For this you need nine pieces of paper numbered 1-9.
Investigate different ways of placing the numbers on the diagram to make equivalent fractions.

Start by keeping one of the denominators the same and changing the other 3 numbers to make equivalent fractions.



Example - Find the equivalent fraction: $\frac{3}{8} = \frac{\quad}{40}$

The denominator has been multiplied by 5.

We multiply the numerator by the same value:

$$\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

The missing number is 15.

Practice

Complete these pairs of equivalent fractions

a) $\frac{1}{6} = \frac{\quad}{18}$ **b)** $\frac{3}{7} = \frac{\quad}{14}$ **c)** $\frac{3}{8} = \frac{\quad}{48}$

d) $\frac{5}{6} = \frac{\quad}{36}$ **e)** $\frac{4}{9} = \frac{24}{\quad}$ **f)** $\frac{5}{7} = \frac{\quad}{56}$

g) $\frac{8}{9} = \frac{40}{\quad}$ **h)** $\frac{7}{12} = \frac{84}{\quad}$ **i)** $\frac{7}{8} = \frac{49}{\quad}$

5.4 Simplifying Fractions

We can **simplify** a fraction by finding **common factors** of the numerator and the denominator.

Example - Write $\frac{32}{56}$ in its simplest form.

32 = 4 x 8 and 56 = 7 x 8. 8 is a common factor.

$$\frac{32}{56} = \frac{4 \times 8}{7 \times 8} = \frac{4}{7}$$

Cancelling the 8s gives the answer.

$\frac{4}{7}$ is $\frac{32}{56}$ written in its **lowest terms.**

Example - What fraction of 1 m is 35 cm?

100 cm = 1m, so 35 cm = $\frac{35}{100}$ m = $\frac{7}{20}$ m

Remember to simplify your answer!

Practice

i. Simplify the fractions:

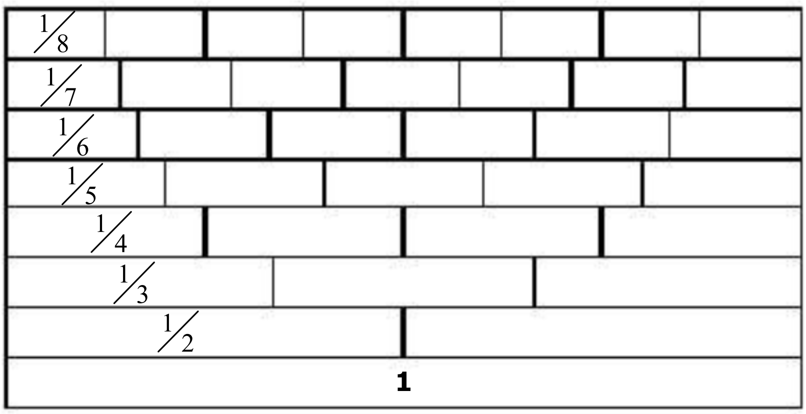
a) $\frac{6}{10}$ **b)** $\frac{12}{18}$ **c)** $\frac{42}{66}$ **d)** $\frac{132}{144}$ **e)** $\frac{54}{162}$ **f)** $\frac{49}{77}$

ii. Write the answer in its lowest terms:

a) What fraction of a kilogram is 24 grams? **b)** How many days are there in 60 hours?

5.5 Ordering Fractions

We can use a **fraction wall** to compare fractions with different denominators.



Think

Look at the wall. Which fraction is bigger, $\frac{1}{2}$ or $\frac{1}{3}$? How about $\frac{1}{4}$ and $\frac{1}{7}$?

Is this statement true: $\frac{3}{4} > \frac{2}{3}$. How can you use the wall to find out?

We can order sets of fractions by writing them all with a **common denominator**. A set of fractions has a common denominator if their denominators are all the same.

Example - Order $\frac{1}{2}, \frac{7}{10}, \frac{3}{4}, \frac{3}{5}$

Find a common denominator for the fractions:

$$\frac{1}{2} = \frac{1\times10}{2\times10} = \frac{10}{20}, \ \frac{7}{10} = \frac{7\times2}{10\times2} = \frac{14}{20}, \ \frac{3}{4} = \frac{3\times5}{4\times5} = \frac{15}{20}, \ \frac{3}{5} = \frac{3\times4}{5\times4} = \frac{12}{20}$$

By comparing the numerators we see that the order is:

$$\frac{10}{20}, \frac{12}{20}, \frac{14}{20}, \frac{15}{20} \text{ or } \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{3}{4}.$$

20 is a common denominator for these fractions.

Practice

i. Use the fraction wall to write the correct symbol > or < between the two fractions:

a) $\frac{1}{4} \quad \frac{1}{7}$ **b)** $\frac{1}{2} \quad \frac{7}{10}$ **c)** $\frac{5}{6} \quad \frac{3}{8}$ **d)** $\frac{3}{7} \quad \frac{32}{56}$

ii. Find which fraction is bigger by writing them with common denominators:

a) $\frac{3}{4}$ or $\frac{5}{6}$ **b)** $\frac{9}{11}$ or $\frac{7}{9}$ **c)** $\frac{5}{6}$ or $\frac{3}{8}$ **d)** $\frac{5}{7}$ or $\frac{7}{9}$

iii. Order each set of fractions. Start with the smallest:

a) $\frac{17}{28}, \frac{3}{4}, \frac{11}{14}, \frac{5}{7}$ **b)** $\frac{7}{12}, \frac{2}{3}, \frac{17}{24}, \frac{3}{4}$ **c)** $\frac{13}{20}, \frac{3}{4}, \frac{4}{10}, \frac{5}{8}$

5.6 Changing Fractions to Decimals

We can write fractions as decimals by finding an equivalent fraction with a denominator of 10, 100, 1000....

Example - Write $\frac{2}{5}$ as a decimal.

$\frac{2}{5} = \frac{4}{10} =$ four tenths $= 0.4$.

Example - Write $3\frac{3}{4}$ as a decimal.

$\frac{3}{4} = \frac{75}{100} =$ seventy five hundredths

= seven tenths and five hundredths = 0.75.

$3\frac{3}{4} = 0.75$

Example - Write $\frac{3}{8}$ as a decimal.

$\frac{3}{8}$ means $3 \div 8$ and $8\overline{)3.000}$ = 0.375.

So, $\frac{3}{8} = 0.375$.

We use division because we cannot write the fraction with a denominator of 10, 100.....

Think

We can also write decimals as fractions. Can you write 0.2, 0.75 and 0.05 as fractions?

Practice

i. Write each fraction as a decimal:

a) $\frac{2}{5}$ **b)** $\frac{1}{4}$ **c)** $2\frac{4}{25}$ **d)** $\frac{5}{16}$ **e)** $1\frac{7}{8}$

ii. Write each pair of fractions as decimals and put the correct symbol > or < between them:

a) $\frac{2}{5} \quad \frac{1}{2}$ **b)** $\frac{8}{10} \quad \frac{3}{4}$ **c)** $\frac{8}{10} \quad \frac{22}{25}$ **d)** $\frac{3}{20} \quad \frac{2}{25}$

iii. Write each decimal as a fraction. Write the fraction in its lowest terms:

a) 0.3 **b)** 0.42 **c)** 2.78 **d)** 7.12 **e)** 0.325

iv. Order each set by writing them as decimals:

a) $\frac{1}{2}$, 0.26, $\frac{1}{4}$, 0.3, $\frac{31}{100}$ **b)** 0.91, $\frac{4}{5}$, $\frac{19}{20}$, 0.85 **c)** $1\frac{3}{4}$, 1.35, $1\frac{7}{10}$, 1.4, $1\frac{16}{20}$

v. Order each set of measurements, starting with the largest:

a) 4.101 m, 4.009 m, 4.0059 m

b) $\frac{1}{8}$ kg, 0.55 kg, 0.525 kg, 1.25 kg

c) $9\frac{9}{10}$ tonnes, 9.904 tonnes, 9.804 tonnes, 9.99 tonnes

d) 2.02 litres, $\frac{1}{5}$ litres, 2.0 litres, 0.022 litres

5.7 Fractions as Recurring Decimals

Think

In the above example we used division to show that $\frac{3}{8} = 0.375$. Use the same method to change $\frac{2}{3}$ and $\frac{2}{7}$ into decimals. Write your answers below to 12 decimal places. What do you notice?

A number or group of numbers that repeats like this is called a **recurring decimal**.

We use a dot above the figures that **recur**:

$$\frac{2}{3} = 0.66666666 = 0.\dot{6} \qquad \frac{2}{7} = 0.285714285714 = 0.\dot{2}8571\dot{4}$$

Practice

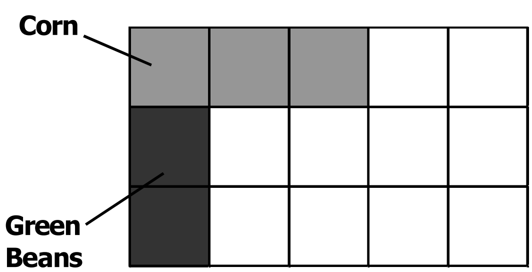
Write the fractions as recurring decimals. Use dots to show how each decimal recurs

a) $\frac{4}{9}$ **b)** $\frac{2}{11}$ **c)** $\frac{7}{9}$ **d)** $\frac{8}{7}$

5.8 Adding and Subtracting Fractions

Think

This week Myint San planted Betel nut in four squares of her garden. Shade this area on the diagram. What fraction of her garden is now planted? Express this in its simplest form.



Fractions with the same denominator are added or subtracted by adding or subtracting the numerators. If fractions have different denominators, then we write them as equivalent fractions.

Example - $\frac{5}{15} + \frac{4}{15} = \frac{9}{15} = \frac{3}{5}$

$\frac{8}{15} - \frac{3}{15} = \frac{5}{15} = \frac{1}{5}$

Example - $\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{7}{15}$

To add or subtract mixed numbers we follow these steps:

1. Change to improper fractions
2. Change to equivalent fractions with a common denominator.
3. Add or subtract.
4. Write the answer in its simplest form.

Practice

i. Solve and write the answer in its lowest terms

a) $\frac{3}{5} + \frac{1}{5}$ **b)** $\frac{5}{8} + \frac{2}{8}$ **c)** $\frac{5}{16} + \frac{7}{16}$ **d)** $\frac{33}{99} + \frac{11}{99} + \frac{22}{99} + \frac{33}{99}$

ii. The following fractions have different denominators. Solve and write the answer in its lowest terms

a) $\frac{2}{5}+\frac{1}{6}$ **b)** $\frac{3}{7}+\frac{1}{6}$ **c)** $\frac{3}{10}+\frac{2}{3}$ **d)** $\frac{3}{11}+\frac{5}{9}$ **e)** $\frac{3}{8}+\frac{7}{16}$ **f)** $\frac{3}{12}+\frac{1}{6}$

g) $\frac{5}{12}+\frac{1}{6}+\frac{1}{3}$ **h)** $\frac{3}{10}+\frac{1}{5}+\frac{1}{4}$

iii. Subtract and write the answer in its lowest terms

a) $\frac{8}{9}-\frac{2}{9}$ **b)** $\frac{3}{4}-\frac{1}{4}$ **c)** $\frac{2}{3}-\frac{3}{7}$ **d)** $\frac{8}{11}-\frac{2}{5}$ **e)** $\frac{8}{13}-\frac{1}{2}$ **f)** $\frac{66}{99}-\frac{22}{99}-\frac{33}{99}-\frac{11}{99}$

iv. Solve and write the answer in its lowest terms

a) $\frac{7}{12}-\frac{1}{6}+\frac{1}{3}$ **b)** $\frac{5}{8}-\frac{21}{40}+\frac{2}{5}$ **c)** $\frac{3}{8}+\frac{7}{16}-\frac{3}{4}$ **d)** $\frac{13}{16}-\frac{1}{8}+\frac{3}{4}$

v. Follow the steps above to add or subtract the mixed numbers

a) $1\frac{1}{8}+\frac{3}{8}$ **b)** $2\frac{7}{10}-\frac{3}{10}$ **c)** $1\frac{3}{4}+2\frac{3}{4}$ **d)** $3\frac{1}{5}-2\frac{2}{5}$

e) $2\frac{1}{4}+3\frac{1}{2}$ **f)** $5\frac{5}{9}-4\frac{1}{3}$ **g)** $8\frac{4}{7}-2\frac{1}{3}$

vi. Solve these problems.

a) Tin Tin adds two fractions. How many mistakes did he make? Explain the mistakes.

$$\frac{1}{4}+\frac{2}{5}=\frac{5}{20}+\frac{4}{20}=\frac{9}{40}$$

b) Every day Saw Saw spends $\frac{1}{3}$ of his time sleeping, $\frac{1}{4}$ of his time at school and $\frac{1}{12}$ of his time eating. What fraction of the day is remaining?

c) Myint San's garden is $6\frac{1}{4}$ m wide and $5\frac{3}{4}$ m long. What is the perimeter of her garden?

d) At the end of the rainy season she harvested her crops. She had $5\frac{1}{2}$ kg of corn, $2\frac{1}{4}$ kg of green beans and 7 kg of cucumber. What was the total weight of her harvest? Write the answer as a fraction and as a decimal.

Activity - Egyptian Fractions

A unit fraction is a fraction with a denominator of 1, $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{7}$ are unit fractions.

The Egyptians only used unit fractions. The Egyptians wrote $\frac{7}{20}$ by adding unit fractions:

$$\frac{7}{20}=\frac{4}{20}+\frac{2}{20}+\frac{1}{20}=\frac{1}{5}+\frac{1}{10}+\frac{1}{20}$$

Work with a partner to write $\frac{13}{20}$ as the sum of unit fractions. Write some more proper fractions as the sums of unit fractions. Can you calculate $\frac{7}{20}+\frac{13}{40}$ using the Egyptian method?

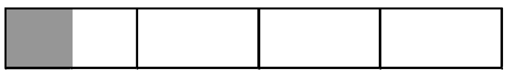
5.9 Multiplying Fractions

We multiply fractions by multiplying the numerators and denominators:

$$\frac{1}{2}\times\frac{1}{4}=\frac{1\times 1}{2\times 4}=\frac{1}{8}$$

Look at the diagram. The shaded region is $\frac{1}{2}$ of $\frac{1}{4}$ the shape.

It is also $\frac{1}{8}$ of the total shape. We see that $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$.



We have $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ and $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$. We learn that 'of' means 'multiplied by'.

This helps us find fractions of whole numbers and quantities.

Example - Find three fifths of 95 metres.

$$\frac{3}{5} \text{ of } 95 = \frac{3}{5} \times 95 = \frac{3}{5} \times \frac{95}{1} = \frac{3}{5 \times 1} \times \frac{5 \times 19}{1}$$

$$= \frac{3}{1} \times \frac{19}{1} = 3 \times 19 = 57 .$$

So, three fifths of 95 metres is 57 metres.

Notice that we can write 95 as a fraction with a denominator of 1 to make the calculation easier:

$$95 = \frac{95}{1} = 95 \div 1 = 95$$

We can cancel the 5's.

Practice

i. Draw diagrams to show that

a) $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ **b)** $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ **c)** $\frac{2}{3}$ of $\frac{1}{3} = \frac{2}{9}$ **d)** $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$

ii. Find

a) $\frac{1}{3}$ of 18 **b)** $\frac{1}{5}$ of 30 **c)** $\frac{3}{8}$ of 64 **d)** $\frac{3}{5}$ of 20 metres

e) $\frac{1}{4}$ of 200 baht **f)** $\frac{1}{7}$ of 1 week **g)** $\frac{3}{7}$ of 35 km **h)** $\frac{7}{8}$ of 1 day (24 hours)

When we multiply two fractions we look for common factors of the numerators and denominators.

Example - Find $\frac{4}{25} \times \frac{15}{16}$

We can cancel 4 with 16 and 5 with 25 to get:

$$\frac{4}{25} \times \frac{15}{16} = \frac{1}{25} \times \frac{15}{4} = \frac{1}{5} \times \frac{3}{4} = \frac{1 \times 3}{5 \times 4} = \frac{3}{20}$$

Example - Find $\frac{3}{5} \times \frac{15}{16} \times \frac{4}{7}$

We cancel 5 with 15 and 16 with 4 to get:

$$\frac{3}{5} \times \frac{15}{16} \times \frac{4}{7} = \frac{3}{1} \times \frac{3}{16} \times \frac{4}{7} = \frac{3}{1} \times \frac{3}{4} \times \frac{1}{7} = \frac{3 \times 3 \times 1}{1 \times 4 \times 7} = \frac{9}{28}$$

Practice

i. Find

a) $\frac{3}{4} \times \frac{1}{2}$ **b)** $\frac{2}{3} \times \frac{5}{7}$ **c)** $\frac{1}{2} \times \frac{7}{8}$ **d)** $\frac{3}{4} \times \frac{1}{5}$ **e)** $\frac{1}{7} \times \frac{3}{5}$

ii. Cancel the common factors to find

a) $\frac{7}{8} \times \frac{4}{21}$ **b)** $\frac{3}{4} \times \frac{16}{21}$ **c)** $\frac{48}{55} \times \frac{5}{12}$ **d)** $\frac{4}{15} \times \frac{25}{64}$ **e)** $\frac{3}{7} \times \frac{28}{33}$

f) $\frac{3}{7} \times \frac{5}{9} \times \frac{14}{15}$ **g)** $\frac{15}{16} \times \frac{8}{9} \times \frac{4}{5}$ **h)** $\frac{3}{10} \times \frac{5}{9} \times \frac{6}{7}$ **i)** $\frac{7}{16} \times \frac{8}{21} \times \frac{9}{11}$

iii. The following questions contain mixed numbers. Solve by changing them to improper fractions and then cancelling common factors

a) $1\frac{2}{5} \times \frac{2}{5}$ **b)** $1\frac{1}{4} \times \frac{2}{5}$ **c)** $3\frac{1}{3} \times 2\frac{2}{5}$ **d)** $3\frac{1}{2} \times 4\frac{2}{3}$ **e)** $6\frac{2}{5} \times 1\frac{7}{8} \times \frac{7}{12}$

5.10 Dividing by Fractions

Dividing by fractions is quite easy. First we **invert** the fraction and then we multiply.

Example - Find $5 \div \frac{1}{3}$

$$5 \div \frac{1}{3} = 5 \times \frac{3}{1} = 5 \times 3 = 15$$

Example - Find $\frac{7}{16} \div \frac{5}{8}$

$$\frac{7}{16} \div \frac{5}{8} = \frac{7}{16} \times \frac{8}{5} = \frac{7}{8 \times 2} \times \frac{8 \times 1}{5} = \frac{7}{2} \times \frac{1}{5} = \frac{7}{10}$$

Practice

Find

a) $8 \div \frac{4}{5}$ **b)** $18 \div \frac{6}{7}$ **c)** $35 \div \frac{5}{7}$ **d)** $44 \div \frac{4}{9}$ **e)** $36 \div \frac{4}{7}$

f) $\frac{21}{32} \div \frac{7}{8}$ **g)** $\frac{3}{56} \div \frac{9}{14}$ **h)** $\frac{21}{22} \div \frac{7}{11}$ **i)** $\frac{9}{26} \div \frac{12}{13}$

Example - Find $3\frac{1}{8} \div 8\frac{3}{4}$

Step 1: Change mixed numbers to improper fractions. $3\frac{1}{8} \div 8\frac{3}{4} = \frac{25}{8} \div \frac{35}{4}$,

Step 2: Invert the second fraction. $\frac{25}{8} \div \frac{35}{4} = \frac{25}{8} \times \frac{4}{35}$,

Step 3: Cancel the common factors $\frac{25}{8} \times \frac{4}{35} = \frac{5 \times 5}{4 \times 2} \times \frac{4 \times 1}{5 \times 7} = \frac{5}{2} \times \frac{1}{7}$,

Step 4: Find the answer! $\frac{5}{2} \times \frac{1}{7} = \frac{5}{14}$

Practice

i. Use the steps in the example to find

a) $5\frac{4}{9} \div \frac{14}{27}$ **b)** $4\frac{2}{7} \div \frac{9}{14}$

c) $3\frac{1}{8} \div 3\frac{3}{4}$ **d)** $7\frac{1}{5} \div 1\frac{7}{20}$

ii. Look at the square. Each letter a,b,c,d is a proper fraction in its lowest terms.
Use this information to find a,b,c,d:

a + b = $\frac{3}{4}$ a x c = $\frac{1}{8}$

d = 3 x b a x d = $\frac{3}{8}$

a	**b**
c	**d**

Activity - What steps are needed to find the answer to this problem? $2\frac{1}{4} \times \frac{3}{14} \div 1\frac{2}{7}$

Write the steps needed. Compare what you have written with your partner.
When you have the correct steps solve the problem together.

Step 1:

Step 2:

Step 3:

Step 4:

6. Percentages

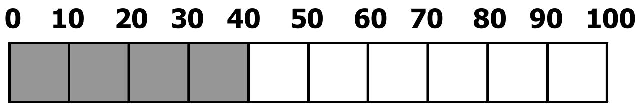
6.1 Introduction

Cent is a Latin word for one hundred. **Percent** means per hundred. If 60 per cent of workers in a factory are women then 60 out of every 100 workers are women. We use the symbol % when writing about **percentages**, 60 per cent = 60 %.

Think

60 % of workers in a factory are women. There are 200 workers. How many women are there?

6.2 Percentages, Fractions and Decimals

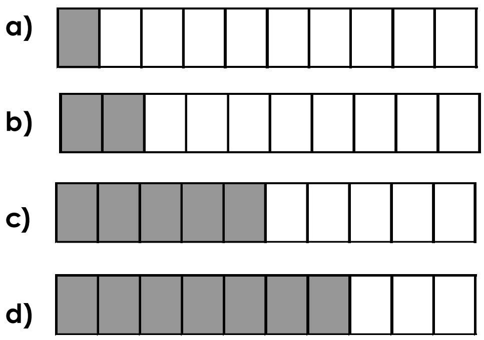


In the diagram 40 parts out of 100 are shaded. As a fraction this is $\frac{40}{100}$. As a decimal this is 0.4.

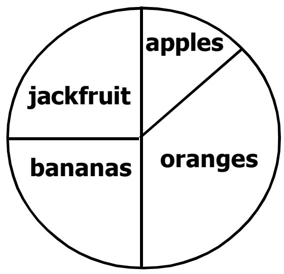
It is also 40 %. We see that $\frac{40}{100} = 0.4 = 40\ \%$.

Think

i. Write the shaded part of each diagram as a percentage, a fraction and a decimal.



ii. A class of students was asked which fruit they liked best. The results are in the chart below.



Estimate:
a) The percentage of students who chose bananas.
b) The percentage who chose apples.
c) The percentage that did not choose jackfruit.

We can write fractions and decimals as percentages.

Example - Write $\frac{7}{20}$ as a percentage.

$\frac{7}{20} = \frac{7}{20} \times 100\ \% = 35\ \%$

Example - Write 0.7 and 1.24 as percentages.

0.7 = 0.7 x 100 % = 70 %
1.24 = 1.24 x 100 % = 124 %

Think

We can also write percentages as fractions and decimals. Try to write 30 % and 62.5 % as fractions and as decimals.

Example - Write $12\frac{1}{2}$ % as a fraction.

$$12\frac{1}{2}\ \% = 12.5\ \% = \frac{12.5}{100}$$ Change to a fraction by dividing by 100.

$$= \frac{12.5 \times 10}{100 \times 10} = \frac{125}{1000}$$ Multiply the numerator and denominator by 100 to get rid of the decimal point.

$$= \frac{1 \times 125}{8 \times 125} = \frac{1}{8}$$ Simplify the answer.

Practice

i. Write as fractions in their lowest terms

a) 30 % **b)** 85 % **c)** 42.5 % **d)** 5.25 %

ii. Write as decimals

a) 44 % **b)** 68 % **c)** 170 % **d)** $16\frac{1}{2}\%$ **e)** $28\frac{2}{5}\%$ **f)** $235\frac{3}{4}\%$

iii. Complete the table. The first row is completed as an example

	Fraction	Decimal	Hundredths	Percentage
a)	$\frac{1}{10}$	0.1	$\frac{10}{100}$	10 %
b)	$\frac{\ }{10}$	0.	$\frac{\ }{100}$	90 %
c)	$\frac{\ }{10}$	0.8	$\frac{\ }{100}$	%
d)	$\frac{\ }{10}$	0.	$\frac{60}{100}$	%
e)	$\frac{3}{10}$	0.	$\frac{\ }{100}$	%
f)	$\frac{1}{2}$	0.	$\frac{\ }{100}$	%
g)	$\frac{1}{\ }$	0.	$\frac{25}{100}$	%
h)	$\frac{3}{4}$	0.	$\frac{\ }{100}$	%
i)		0.2		

6.3 Percentages and Quantities

We can write one **quantity** as a percentage of another. First we divide the first quantity by the second. Then we multiply the fraction by 100 %.

Example - Find 4 as a percentage of 20.

$$\frac{4}{20} = \frac{4}{20} \times 100\,\% = 20\,\%$$

Example - Write 20 cm as a percentage of 3 m.

3 m = 3 x 100 cm = 300 cm. So,

$$\frac{20}{300} \times 100\,\% = \frac{20}{3}\,\% = 6\frac{2}{3}\,\%$$

Practice

Write the first quantity as a percentage of the second.

a) 3, 12
b) 15, 20
c) 60 cm, 4 m
d) 600 m, 2 km
e) 1200 g, 2 kg
f) 0.01 m, 150 cm
g) 50 kyat, 100 kyat
h) 135 kyat, 450 kyat

i) In her Maths exam, Thanda scored 28 out of 40. What was her mark as a percentage?

j) Mi Mi earns 800 kyat a day working in a factory. She spends 250 kyat on food, 100 kyat on a drink and saves the remainder. What percentage of her money does she save?

We can also find a percentage of one quantity.

Example - Find 12 % of 450.

$12\ \% = \frac{12}{100}$. So,

$12\ \%$ of $450 = \frac{12}{100} \text{x } 450 = 0.12 \text{ x } 450 = 54.$

Example - Find $7\frac{1}{3}\%$ of 6 m.

$7\frac{1}{3}\%$ of 6 m = $7\frac{1}{3}\%$ of 600 cm

$= \frac{22}{3}\%$ of 600 cm $= \frac{22}{3\times100}$ x 600 cm

$= \frac{22}{300}\times 600\,\text{cm} = 22\times 2\,\text{cm} = 44\,\text{cm}$

Practice

Calculate the following:

a) 40 % of 120
b) 70 % of 360
c) 80 % of 1150 g
d) 63 % of 4 m
e) 17 % of 2000 m
f) 12 % of 400 kyat
g) $5\frac{1}{4}\%$ of 56 mm

h) There are 120 shops in Max Shopping centre. 35 % of the shops sell clothes. How many shops sell food? How many shops do not sell food?

i) It is estimated that 62 % of the people in Putao are Kachin. If there are 150,000 in total, how many are Kachin?

6.4 Percentage Increase and Decrease

If we **increase** a quantity by 30 %, the increased quantity is (100 + 30) % = 130 % of the original.
If we **decrease** a quantity by 30 %, the decreased quantity is (100 - 30) % = 70 % of the original.

Example - Increase 180 by 30 %

The new number is 130 % of the original.

$130\ \%$ of $180 = \frac{130}{100}\times180 = \frac{13}{10}\times180 = 13\times18 = 234$

The new value is 234.

Example - Decrease 180 by 30 %

The new value is 70 % of the original.

$70\ \%$ of $180 = \frac{70}{100}\times180 = \frac{7}{10}\times180 = 7\times18 = 126$

The new value is 126.

Practice

i. Increase:

a) 100 by 40 % **b)** 340 by 60 % **c)** 1600 by 73 % **d)** 145 by 120 %

ii. Decrease

a) 100 by 30 % **b)** 350 by 40 % **c)** 3400 by 28 % **d)** 250 by $37\frac{1}{5}\%$

iii. Solve the following:

a) The population of the world is about 6500 million. By 2050 scientists think the population will have increased by about 40 %. What will the world's population be in 2050?

b) The owner of a clothes shop in Lashio offers a **discount** on the clothes in her shop. (Discount is money taken away from the price of something).
The table below shows the original price (before the discount) of the clothes and the per centage discount. Complete the table by calculating the new prices.

Type of clothing	Original price	Discount	New price
Jeans	9000 kyat	35 %	
T-Shirt	2500 kyat	25 %	
Sandals	1750 kyat	10 %	
Jacket	13000 kyat	50 %	
Shoes	5500 kyat	20 %	
Shirt	4250 kyat	15 %	

6.5 Finding Percentage Increases and Decreases

Example - Naw Cleo bought a motorbike for 40 lak. She sold it one year later for 34 lak. What is the percentage decrease in the price?

Loss = 40000 - 34000 = 6000 — First calculate the difference between the two prices.

$6000 = \frac{6000}{40000}$ of 40000. — Then calculate the difference as a fraction of the original price.

$\frac{6000}{40000} = \frac{6}{40} = 0.15 = 15\ \%$ — Change this fraction to a percentage to get the percentage decrease in price.

Think

In 1984 there were 10,000 refugees living on the Thai-Burma border. By 1994 the number had increased to 80,000. Can you calculate the percentage increase in the numbers of refugees?

Practice

i. Find the percentage price increases

a) Buying price 120 kyat, selling price 150 kyat

b) Buying price 160 kyat, selling price 200 kyat

c) Buying price 550 kyat, selling price 605 kyat

ii. Find the percentage price decreases

a) Buying price 200 kyat, selling price 160 kyat

b) Buying price 1250 kyat, selling price 1000 kyat

c) Buying price 640 kyat, selling price 160 kyat

iii. The shop owner in Mergui offers a discount on more items in the shop. Find the percentage decrease on the items.

Type of clothing	Original price	Discount	New price
Longyi	2000 kyat		1200 kyat
Earrings	5000 kyat		2000 kyat
Socks	2000 kyat		1200 kyat
Skirt	3500 kyat		2450 kyat
Underpants	3500 kyat		1400 kyat

6.6 Compound Percentage Problems

Sometimes a percentage increase or decrease happens more than once.

Example - Htaw Pai bought a motorbike for 350,000 kyat. The value of the bike decreased by 10 % each year What is the value of the bike after 3 years?

After 1 year the value of the bike is $\frac{90}{100}$ x 350,000 = 315,000

After 2 years the value of the bike is $\frac{90}{100}$ x 310,500 = 283,500

After 3 years the value of the bike is $\frac{90}{100}$ x 283,500 = 255,150

In the above example we calculated the answer in 3 steps. We also use this formula to calculate the answer in one step:

Value after n years = original value x multiplier^n

In this example multiplier = $\frac{90}{100}$ = 0.9

Practice

i. Use the formula to find the value of Htaw Bai's motorbike after 5 years. (Use the fact that 0.9^5 = 0.59 to 2 decimal places).

The following two problems deal with compound percentage *increases*

ii. The population of Israel is 7,000,000 to the nearest million. This figure is increasing by around 2 % each year. What will be the population of Israel in 20 years? (Use the fact that 1.02^{20} = 1.5 to 1 decimal place).

iii. China uses 6.6 million barrels of oil everyday. The amount the country uses has increased by an average of 7 % per year since 1990. If this growth continues how many barrels of oil will China use per day in 5 years? (Use the fact that 1.07^5 = 1.4 to 1 decimal place).

7. Ratio

7.1 Introduction

We use **ratios** to compare **related** quantities.

Example - Nang Hseng and Sai Lek have 18 grandchildren: 10 grandsons and 8 granddaughters. The ratio of the number of grandsons to the total number of grandchildren is 10 to 18.

We can write this as a fraction: $\dfrac{\text{Number of grandsons}}{\text{Total number of grandchildren}} = \dfrac{10}{18} = \dfrac{5}{9}$

or as a ratio: Number of grandsons : total number of grandchildren = 10 : 18 = 5 : 9.
We simplify ratios in the same way as we simplify fractions.

Think

a) What is the ratio of the number of granddaughters to the number of grandchildren? Simplify your answer.
b) What is the ratio of the number of granddaughters to the number of grandsons?

7.2 Simplifying Ratios

Example - Simplify the ratio 2 cm to 1 m.
Before we compare, the quantities must have the same units.
2 cm : 1 m = 2 cm : 100 cm = 1 : 50
We do not need to write the units but we say, 'the ratio of 2 cm to 1 m is 1 to 50'.

Example - Simplify the ratio 24 to 72.
24 : 72 = 3 x 8 : 9 x 8 = 3 : 9
= 1 x 3 : 3 x 3 = 1 : 3

Example - Simplify the ratio $2 : \frac{1}{3}$
Here we use multiplication:
$2 : \frac{1}{3} = 2 \times 3 : \frac{1}{3} \times 3 = 6 : 1$

Example - Simplify the ratio $\frac{2}{3} : \frac{1}{5}$
We need to multiply by the lowest common multiple to make whole numbers
$\frac{2}{3} : \frac{4}{5} = 15 \times \frac{2}{3} = 15 \times \frac{4}{5}$
$= 5 \times 2 : 3 \times 4 = 10 : 12 = 5 : 6$

Practice

i. Simplify the ratios:

a) 8 : 10 **b)** 12 : 18 **c)** 32 : 96 **d)** 12 cm to 2 m **e)** 4 : 6 : 10

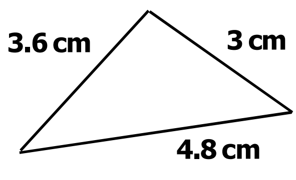
f) 144 : 12 : 24 **g)** 7 : 56 : 49 **h)** $5 : \frac{1}{3}$ **i)** $\frac{1}{3} : \frac{3}{4}$ **j)** $\frac{5}{4} : \frac{6}{7}$ **k)** $\frac{1}{6} : \frac{1}{8} : \frac{1}{12}$

ii. Solve the following:

a) Moe Aye walks 2 km to school in 40 minutes. Nai Aung cycles 5 km to school in 15 minutes. What is the ratio of i) Moe Aye's distance to Nai Aung's distance and ii) Moe Aye's time to Nai Aung's time.

b) Earlier we learnt that Naw Kaw's harvest was $5\frac{1}{2}$ kg of corn, $2\frac{1}{4}$ kg of green beans and 7 kg of cucumber. Find the ratio of the amount of vegetables to one another.

c) Find the ratio of the sides of the triangle to one another.



7.3 Finding Quantities

Example - Find the missing number: __ : 4 = 3 : 5

Fill the gap with an X, $X : 4 = 3 : 5$.

Write as fractions, $\frac{X}{4} = \frac{3}{5}$

Multiply by 4, $4 \times \frac{X}{4} = 4 \times \frac{3}{5}$

This gives $X = \frac{12}{5} = 2\frac{2}{5}$

So, $2\frac{2}{5} : 4 = 3 : 5$

Example - Two distances are in the ratio 12 : 5. The first distance is 8 km. Find the second distance.

Let X be the second distance, $8 : X = 12 : 5$.

So, $\frac{X}{8} = \frac{5}{12}$.

Multiply by 8, $8 \times \frac{X}{8} = 8 \times \frac{5}{12}$

This gives $X = \frac{10}{3} = 3\frac{1}{3}$

So the second distance is $3\frac{1}{3}$ km

Practice

i. Find the missing numbers:

a) 2 : 5 = 4 : **b)** : 6 = 12 : 18 **c)** 9 : 6 = : 4 **d)** : 9 = 3 : 5
e) 9 : 5 = : 4 **f)** : 3 = 5 : 2 **g)** 3 : = 5 : 1 **h)** : 6 = 5 : 8

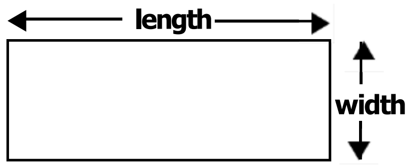
ii. Find the second quantity:

a) Two distances are in the ratio 12 : 8. The first distance is 8 km. Find the second distance.

b) If the ratio in question a) was 8 : 12, what would the second distance be?

c) The ratio of Shine's money to Htoo Chit's money is 8 : 10. Htoo Win has 25 dollars. How much money does Shine have?

d) In the rectangle, the ratio of length to width is 9 : 4. The length is 24 cm. What is the width?



7.4 Sharing Quantities

Example - Share 60 baht between Hser Moo and Hsa Say in the ratio 3 : 2.

number of shares needed:	**3 + 2 = 5**
value of 1 share:	**60 ÷ 5 = 12**
value of 2 shares:	**2 x 12 + 24**
value of 3 shares:	**3 x 12 = 36**

Hser Moo gets 3 shares, which is 36 baht. Hsa Say gets 2 shares, which is 24 baht.

Think

Look at the amounts that Min Min and Bo Aung received. How do we know if the answer is right? How much would they get if the ratio was 2 : 4?

Practice

a) Share 80 kyat in the ratio 3 : 2
b) Share 32 kyat in the ratio 3 : 5
c) Share 45 kyat in the ratio 4 : 5
d) Divide 26 kyat in the ratio 4 : 5 : 4

7.5 Map Scales

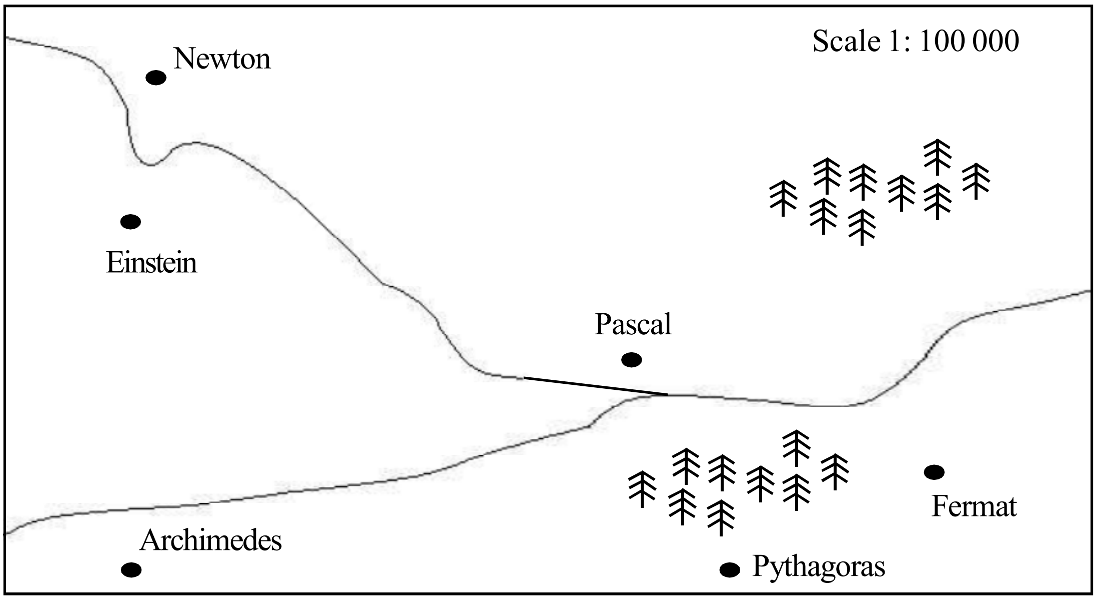
We use map scales to measure real distances between places. Look at the map on the next page. The scale is 1 : 100 000. This means 1 cm on the map is 100 000 cm in real distances. 100 000 cm = 1000 m = 1 km, so 1 cm on the map is 1 km in real distances.

Activity - The map is of a country named Geekland. Each town is named after a famous mathematician. The distance between Archimedes and Pythagoras on the map is 8 cm. How far is it from Archimedes to Pythagoras in kilometres?

Use a ruler to measure the distance on the map between:

a) Pythagoras and Newton
b) Pascal and Einstein
c) Pascal and Fermat
d) Archimedes and Newton

Now find the real distances using the map scale.



8. Indices

8.1 Positive Indices

In Section 2.3 we learnt about numbers with positive indices.
We learnt how to write in index form:
$3 \times 3 \times 3 \times 3 = 3^4$
And how to find the value of numbers in index form:
$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

We can multiply different powers of the same number by adding the indices:
$2^2 \times 2^3 = 2^{2+3} = 2^5$

Think

a) Complete the statement to write $2^5 \div 2^3$ as a single number in index form:

$2^5 \div 2^3 = \frac{2^5}{\_\_}$ = ________________ =

b) Fill in the missing word and complete the statement:

We can divide different powers of the same number by ______________the indices:

$2^5 \div 2^3 = 2^{\ -\ }$ =

Practice

Write as a single number in index form:

a) $3^5 \times 3^2$
b) $7^5 \times 7^3$
c) $5^4 \times 5^4$
d) $12^4 \times 12^5$
e) $4^4 \div 4^2$
f) $10^8 \div 10^3$
g) $2^3 \div 2^3$
h) $6^{12} \div 6^7$
i) $15^8 \div 15^4$
j) $2^2 \times 2^4 \times 2^3$
k) $4^2 \times 4^3 \div 4^4$
l) $3^6 \div 3^2 \times 3^4$

Look at question **g)**. The answer is $2^3 \div 2^3 = 2^0$.

Also, $2^3 \div 2^3 = \frac{2 \times 2 \times 2}{2 \times 2 \times 2} = 1$.

Any number with an index of zero is equal to one:
$2^3 \div 2^3 = 2^0 = 1.$

8.2 Negative Indices

Example - Find the value of $2^3 \div 2^5$

Subtracting the indices gives:

$$2^3 \div 2^5 = 2^{3-5} = 2^{-2}$$

As a fraction we have:

$$2^3 \div 2^5 = \frac{2^3}{2^5} = \frac{2\times2\times2}{2\times2\times2\times2\times2} = \frac{1}{2^2}$$

The value of $2^3 \div 2^5 = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$\frac{1}{2^2}$ is called the **reciprocal** of 2^2.
Also, 2^2 is called the reciprocal of $\frac{1}{2^2}$.

Practice

i. Find the reciprocal of:

a) 2^4
b) 7^3
c) 7^{-3}
d) 5^{-5}

ii. Write as a single number in index form:

a) $5^2 \div 5^4$
b) $6^4 \div 6^7$
c) $10^3 \div 10^6$
d) $a^5 \div a^7$

iii. Find the value of:

a) 2^{-3}
b) 3^{-3}
c) 7^{-2}
d) 8^{-2}
e) $5^3 \div 5^6$
f) $4^{-1} \times 4^{-2}$

8.3 Standard Form

In Section 4.6, you were asked to guess the distance to the moon. The answer is 400,000 km.
When measuring large numbers like this, scientists use **standard form:**

$$400{,}000 = 4 \times 10 \times 10 \times 10 \times 10 \times 10 = 4 \times 10^5$$

Standard form is a number between 1 and 10 multiplied by a power of 10.

Example - Write 68,000 in standard form.
$68\ 000 = 6.8 \times 10{,}000 = 6.8 \times 10^4$

Example - Write 0.01934 in standard form.
$0.01934 = 1.934 \div 100 = 1.934 \times \frac{1}{100}$
$= 1.934 \times 10^{-2}$

Practice

i. Write the following in standard form:

a) 2500
b) 630
c) 39,070
d) 260,000
e) 4,060,000
f) 80,000,000,000
g) 0.79
h) 0.0048
i) 0.0805
j) 0.08808
k) 0.684
l) 0.000 000 000 073

ii. Write the following as ordinary numbers:

a) 3.78×10^3
b) 1.26×10^{-3}
c) 5.3×10^6
d) 3.76×10^{-6}
e) 4.25×10^{12}
f) 4.43×10^{-8}

8.4 Significant Figures

Think

Ti Reh is 1678 mm tall. How tall is he:

a) to the nearest 10 mm?
b) to the nearest cm?
c) in metres (to 2 d.p.)?
d) in km (to 5 d.p.)?

The numbers 1,6 and 8 appear in each answer. 1, 6 and 8 are called the **significant figures (s.f.).** Each answer is given to 3 significant figures. The first significant figure is 1. The second is 6. The third is 8.

In standard form the first significant figure is the number in the tens column.

Example - Find:
a) the 1st significant figure and
b) the 3rd significant figure
of 0.001503.

In standard form $0.001503 = 1.503 \times 10^{-3}$.
a) The first signifcant figure is 1.
b) The third significant figure is 0.

Practice

Write down the significant figure given in brackets.

a) 36. 2 (1st)
b) 0.0867 (2nd)
c) 3.786 (3rd)
d) 34.807 (4th)
e) 0.07603 (3rd)

Example - Give 32685 correct to 1 s.f.

$32685 = 3.2685 \times 10^4$.

3 is the first s.f. The 2nd s.f. is less than 5, so we round down.

So, $32685 = 3 \times 10^4 = 30000$ to 1 s.f.

Example - Give 0.02186 correct to 3 s.f.

$0.02186 = 2.186 \times 10^{-2}$.

Here the 4th s.f. is greater than 5 so we round up.

So, $0.02186 = 2.19 \times 10^{-2} = 0.0219$ to 3 s.f.

Practice

i. Give the following numbers correct to 1 s.f.

a) 59727 **b)** 476 **c)** 586359 **d)** 26 **e)** 4099 **f)** 4396359

ii. Give the following numbers correct to 2 s.f.

a) 4674 **b)** 58700 **c)** 9973 **d)** 72601 **e)** 444 **f)** 53908

iii. Give the following numbers correct to 3 s.f.

a) 0.008463 **b)** 5.8374 **c)** 46.8451 **d)** 7.5078 **e)** 369.649 **f)** 0.0078547

8.6 Square Roots

We know that $3^2 = 9$. 3 is the square root of 9. In section 3.4 we learnt that $(-3)^2 = 9$. -3 is also the square root of 9. In this module we will study positive square roots.

The square root symbol is $\sqrt{\ }$. We write $\sqrt{9} = 3$.

Example - Find the square root of 0.49.

$0.49 = 0.7^2$. So, $\sqrt{0.49} = 0.7$

Practice

Find the square roots:

a) $\sqrt{9}$ **b)** $\sqrt{25}$ **c)** $\sqrt{64}$ **d)** $\sqrt{1}$ **e)** $\sqrt{4900}$ **f)** $\sqrt{0.64}$ **g)** $\sqrt{0.04}$

In section 2.4 we learnt about prime factors. Prime factors can help us find square roots.

Example - Find the square root of 784.

As a product of prime factors $784 = 2^2 \times 2^2 \times 7^2$.

We can write $2^2 \times 2^2 \times 7^2 = (2 \times 2 \times 7)^2 = 28^2$.

So, $784 = 28^2$ and $\sqrt{784} = \sqrt{28^2} = 28$

Practice

Find the square roots:

a) $\sqrt{576}$ **b)** $\sqrt{1296}$ **c)** $\sqrt{1764}$

The numbers above have whole number square roots. Most numbers do not. If we do not have a calculator, we have to estimate the square root.

Example - Find the square root of 23.

We know that $16 < 23 < 25$. If we take the square roots we have: $\sqrt{16} < \sqrt{23} < \sqrt{25} = 4 < \sqrt{23} < 5$

We learn that $\sqrt{23}$ is between 4 and 5. So, to 1 significant figure $\sqrt{23} = 4$.

Practice

Find the first significant figure of the square roots of these numbers:

a) 17 **b)** 10 **c)** 39 **d)** 79 **e)** 90

8.7 Surds

The area of the square is 20 cm. The length of the side is x cm. So,

$$x^2 = 20$$
$$x = \sqrt{20}$$

If we use a calculator we would find that x = 4.47 to 2 decimal places. To represent the number exactly we can just write $x = \sqrt{20}$. A number written exactly in this form is called a **surd**. Other examples of numbers in surd form are: $\sqrt{3},\ 2-\sqrt{5}, \frac{\sqrt{2}}{2}$

Example - Simplify $\sqrt{12}$

$$\sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$$

Example - Rationalise* $\frac{1}{3\sqrt{5}}$

We need to multiply the top and bottom by $\sqrt{5}$

$$\frac{1}{3\sqrt{5}} = \frac{1}{3\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{3\times 5} = \frac{\sqrt{5}}{15}$$

Example - Rationalise* $\frac{\sqrt{2}}{5+\sqrt{2}}$

Multiply the top and bottom by $(5-\sqrt{2})$

$$\frac{\sqrt{2}}{5+\sqrt{2}} = \frac{\sqrt{2}(5-\sqrt{2})}{(5+\sqrt{2})(5-\sqrt{2})}$$

$$= \frac{5\sqrt{2}-2}{25-5\sqrt{2}+5\sqrt{2}-2} = \frac{5\sqrt{2}-2}{23}$$

* **Rationalise** means to remove the square root in the denominator

Practice

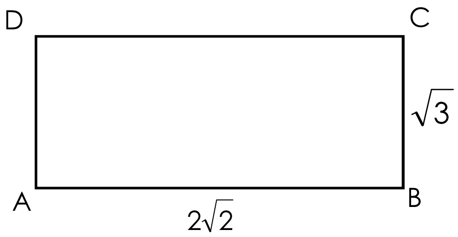
i. a) $\frac{1}{\sqrt{7}}$ **b)** $\frac{1}{\sqrt{17}}$ **c)** $\frac{\sqrt{32}}{\sqrt{2}}$ **d)** $\frac{\sqrt{5}}{3+\sqrt{5}}$ **e)** $\frac{\sqrt{7}}{2-\sqrt{7}}$ **f)** $\frac{\sqrt{3}}{2-\sqrt{3}}$

ii. The lengths of the sides of a rectangle are $3+\sqrt{5}$ and $3-\sqrt{5}$. Find:

a) The perimeter of the rectangle **b)** The area of the rectangle

iii. The rectangle shown has sides $2\sqrt{2}$ and $\sqrt{3}$.

Find the length of the diagonal (using Pythagoras' theorem)



9. Estimation

9.1 Using Significant Figures to Estimate

At the beginning of this module, we learnt how to use rounding to estimate answers.
We can also use significant figures to give estimates.

Example - Correct each number to 1 s.f. and give an estimate:

a) 9.524 x 0.0837

9.524 x 0.0837 ≈ 10 x 0.08 = 8

This symbol means '**approximately** equal to'.

b) $\frac{0.048 \times 3.275}{0.367}$

$$\frac{0.048 \times 3.275}{0.367} \approx \frac{0.05 \times 3}{0.4} = \frac{0.15}{0.4} = \frac{1.5}{4} = 0.4$$

The answer is given to 1 s.f.

Practice

Correct each number to 1 s.f. and give an estimate:

a) 4.78 x 23.7 **b)** 0.0674 x 5.24 **c)** $\frac{3.87 \times 5.24}{2.13}$ **d)** $\frac{0.636 \times 2.63}{5.57}$

Activity - Look at the exercise below and match the calculation with the estimate by correcting each number to 1 significant figure. When you have finished make your own exercise by writing some calculations and corresponding estimates. Give your exercise to your partner to solve.

$\frac{4.257}{83.6}$

64

$\frac{80}{9}$

$0.827 \div 0.093$

$\frac{1}{10}$

$\frac{2.27 \times 3.84}{5.01}$

$(3.827)^3$

2.6

0.532 x 3.621 x 36

72

$\frac{2.93 \times 0.0372}{1.84 \times 0.562}$

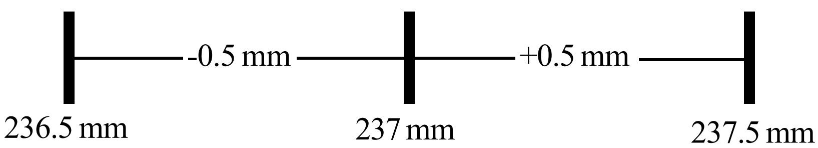
$\frac{1}{20}$

9.2 Upper and Lower Bounds

Consider the measurement:

237 mm, correct to the nearest millimetre

'correct to the nearest millimetre' means that the measurement has been rounded up or down.
The actual length could be anywhere between 236.5 mm and 237.5 mm.



236.5 is the smallest possible value - it is the **lower bound**.
237.5 is the largest possible value - it is the **upper bound**.

Example - Write the lower bound and upper bound for 4.7, correct to 1 decimal place.

The lower bound is 4.65

The upper bound is 4.75

Practice

Write the upper and lower bound for each measurement

i.

a) To the nearest 10: 30, 180, 3020, 10

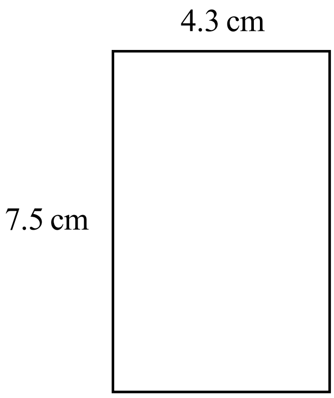
b) To 1 decimal place: 2.9, 13.6, 0.3, 157.5

ii. Complete the table attendances at football stadiums

Stadium	Attendance to the nearest thousand	Lower bound	Upper bound
Manchester United	71,000		
Arsenal	42,000		
West Ham	23,000		
Liverpool	13,000		
Morecambe	1,000		

Example - A rectangle has width 4.3 cm and length 7.5 cm, correct to the nearest 0.1 cm. Find:

a) The lower bounds of the length and width
b) The upper bounds of the length and width
c) The maximum possible area



a) The lower bound for the length is 7.45 cm
The lower bound for the width is 4.25 cm

b) The upper bound for the length is 7.55 cm
The upper bound for the width is 4.35 cm

c) The maximum possible area is 7.55 cm x 4.35 cm = 32.8425 cm^2
(Using the upper bounds)

Practice

i. Two circles have radius r = 1.7, correct to 1 decimal place and R = 31, correct to 3 significant figures.

a) Write down the upper and lower bounds of r and R.

b) Find the smallest possible value of R - r.

ii. The length of each side of a square is 3.7 cm, correct to 2 significant figures.

a) Find the maximum value for the perimeter of the square

b) Find the minimum value for the perimeter of the square

iii. Find the upper bound and lower bound of:

a) The area of a rectangle with sides 6 cm and 8 cm, both correct to the nearest centimetre

b) The perimeter of a square with side 7.5 cm, correct to the nearest centimetre

c) The area of a triangle with base 8 cm and height 6 cm, correct to the nearest centimetre

Glossary of Keywords

Here is a list of Mathematical words from this module. The section where the word appears is given in brackets. Find the words and what they mean - your teacher will test your memory soon!

Whole number	(1.1)
Value	(1.2)
Order	(1.2)
Rounding	(1.3)
Estimate	(1.3)
Round up/Round down	(1.3)
Property	(1.4)
Addition	(1.4)
Subtraction	(1.4)
Commutative Law	(1.4)
Multiplication	(1.5)
Division	(1.5)
Remainder	(1.5)
Operation	(1.6)
Brackets	(1.6)
Powers	(1.6)
Order of operations	(1.6)
Factor	(2.1)
Product	(2.1)
Multiple	(2.1)
Prime Number	(2.2)
Index	(2.3)
Indices	(2.3)
Prime Factor	(2.4)
Highest Common Factor (H.C.F.)	(2.5)
Lowest Common Multiple (L.C.M.)	(2.6)
Postive number	(3.1)
Negative number	(3.1)
Temperature	(3.2)
Degrees Celsius/Centigrade	(3.2)
Expand	(3.4)
Decimal Point	(4.1)
Tenths	(4.1)
Hundredths	(4.1)
Thousandths	(4.1)
Perimeter	(4.2)
Rectangle	(4.2)
Metric Units	(4.4)
Length	(4.4)
Metre (m)	(4.4)
Kilometre (km)	(4.4)
Centimetre (cm)	(4.4)
Millimetre (mm)	(4.4)
Mass	(4.5)
Tonne (t)	(4.5)
Gram (g)	(4.5)
Kilogram (kg)	(4.5)
Pentagon	(4.8)
Decimal Places (d.p.)	(4.9)
Numerator	(5.1)
Denominator	(5.1)
Proper Fraction	(5.1)
Improper Fraction	(5.2)
Mixed Numbers	(5.2)
Equivalent Fractions	(5.3)
Multiplication Wall	(5.3)
Simplify	(5.4)
Common Factors	(5.4)
Cancelling	(5.4)
Lowest Terms	(5.4)
Fraction Wall	(5.5)
Common Denominator	(5.5)
Recurring Decimal	(5.6)
Recur	(5.6)
Invert	(5.10)
Percent	(6.1)
Percentages	(6.1)
Quantity	(6.3)
Increase	(6.4)
Decrease	(6.4)
Discount	(6.4)
Ratio	(7.1)
Related Quantities	(7.1)
Reciprocal	(8.2)
Standard Form	(8.3)
Significant Figures	(8.4)
Approximately	(8.5)
Surd	(8.7)
Rationalise	(8.7)
Lower Bound	(9.2)
Upper Bound	(9.2)

Assessment

This assessment is written to test your understanding of the module. Review the work you have done before taking the test. Good luck!

Part 1 - Vocabulary

These questions test your knowledge of the keywords from this module. Complete the gaps in each sentence by using the words in the box. Be careful, there are 20 words but only 15 questions!

commutative	**equivalent**	**property**	**factor**	**approximately**	**kilometres**	
reciprocal	**perimeter**	**length**	**estimate**	**rationalise**	**numerator**	**recurring**
metres	**brackets**	**multiple**	**operations**	**denominator**	**simplify**	**standard form**

a) 2^{-2} is the ________________ of 2^{2}.

b) We ________________ to remove the square root in the denominator

c) The distance between London and Manchester is measured in ________________.

d) When calculating we expand the ________________ before multiplying.

e) Multiplication and division are ________________.

f) 2 is a ________________ of 18.

g) 18 is a ________________ of 2.

h) The ________________ is the distance all the way around.

i) The ________________ is the bottom number in a fraction.

j) ________________ fractions have the same value.

k) If we ________________ $\frac{9}{24}$ the answer is $\frac{3}{8}$.

l) 0.6666666666666.... is a ________________ decimal.

m) Scientists use ________________ to write large numbers.

n) A proper fraction has a ________________ that is less than the denominator.

o) $\approx$ means ________________ equal to.

Part 2 - Mathematics

These questions test your understanding of the Mathematics in this module. Try to answer all the questions.. Write your calculations and answers on separate paper.

1. Write these numbers as words: **a)** 201 **b)** 1021 **c)** 112354

2. Write these as numbers:
a) seven thousand six hundred and two **b)** thirty two thousand and fifty seven
c) one hundred and eleven thousand

3. Solve:
a) 3021+ 926 **b)** 9217 + 3216 + 824 **c)** 526 - 316 **d)** 1237 - 524
e) Subtract five thousand one hundred and fourty seven from nine thousand three hundred and fourty two.

4. Solve:
a) 798 x 583 **b)** 7281 x 694 **c)** 2050 ÷ 19 **d)** 7974 ÷ 17

5. Solve:
a) 7 + 3 x 2 - 8 ÷ 2 **b)** 19 + 3 x 2 - 4 ÷ 2 **c)** 6 x 8 - 18 ÷ (2 + 4) **d)** 5 + (2 x 10 - 5) - 6

6. **a)** Write down all the multiples of 5 between 19 and 49.
b) Write down all the multiples of 11 between 50 and 100.

7. Which of these numbers are prime numbers:
41, 57, 82, 91, 101, 126, 135

8. Use a factor tree to write each number as a product of prime factors:
a) 24 **b)** 136 **c)** 528

9. Find the Highest Common Factor of:
a) 8, 16 **b)** 36, 48 **c)** 18, 20, 36

10. Find the Lowest Common Multiple of:
a) 6, 16 **b)** 36, 16 **c)** 85, 50

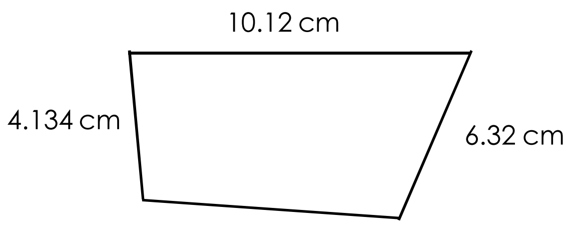
11. Solve:
a) 5 + (-1) - (-3) **b)** 8 - (-7) + 2 **c)** 5 - (6 - 10) **d)** Subtract 7 from -5

12. Complete the statements:
a) A negative number divided by a negative number gives a ______________ answer.
b) A positive number multiplied by a ______________ number gives a negative answer.
c) A ______________ number multiplied by a negative number gives a ______________ answer.

13. Copy and complete the table so that each row, column and diagonal is equal to 18.9.

		8.1
5.4	**6.3**	**7.2**
	10.8	

14. The perimter of the shape is 27 cm. What is the length of the fourth side?



15. Find the following. Write your answer in the units given in brackets:
a) 8 m - 52 cm (cm) **b)** 20 g - 120 mg (mg) **c)** 3.9 m + 582 mm (cm)

16. Solve:

a) 2.16 x 0.082 **b)** 0.081 x 0.32 **c)** 8.2 x 2.8 **d)** 9.8 ÷ 1.4 **e)** 10.24 ÷ 3.2

17. 50 students were asked 'What is your favourite sport?'. The results are in the table.

football	**caneball**	**volleyball**	**total**
15	25	10	50

What fraction chose (write the answer in its lowest terms):

a) football? **b)** caneball? **c)** volleyball?

18. Write these as mixed numbers:

a) $\frac{41}{8}$ **b)** $\frac{67}{5}$

19. Write these as improper fractions:

a) $2\frac{6}{7}$ **b)** $4\frac{7}{9}$

20. Solve the following:

a) $\frac{1}{4}+\frac{7}{10}$ **b)** $\frac{5}{8}-\frac{2}{7}$ **c)** $\frac{4}{5}\times\frac{15}{16}$ **d)** $\frac{28}{27}\div\frac{4}{9}$

21. Write these percentages as fractions in their lowest terms:

a) 45 % **b)** 56.5 % **c)** 9.25 %

22. Calculate the following:

a) 20 % of 200 **b)** 37 % of 9 m

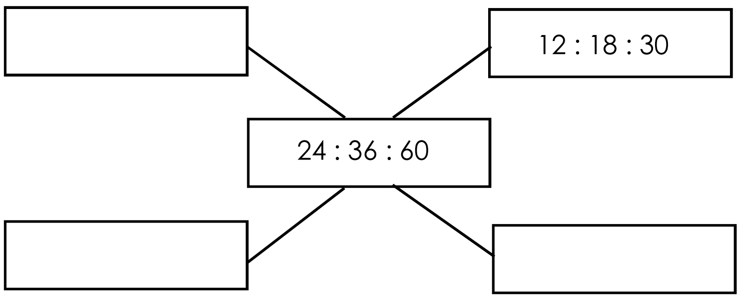
23. Increase:

a) 150 by 50 % **b)** 350 by 20%

24. Decrease:

a) 350 by 40 % **b)** 750 by 13 %

25. Write ratios equivalent to the one in the centre. What is the simplest form of this ratio?



26. Two lengths are in the ratio 3 : 7. The second length is 42 cm. What is the first length?

27. Write as a single number in index form:

a) $5^5 \times 5^5$ **b)** $7^7 \div 7^3$ **c)** $6^3 \div 6^3$

28. Write the following in standard form:

a) 25,000 **b)** 765 **c)** 9,000,000,000

29. Write the following to: 3 significant figures:

a) 0.006758 **b)** 22.8762 **c)** 542.482

30. Rationalise

a) $\frac{3}{\sqrt{5}}$ **b)** $\frac{1}{\sqrt{11}}$ **c)** $\frac{\sqrt{11}}{\sqrt{11}-7}$

31. Write the upper and lower bounds for each measurement

To the nearest 0.5 unit: 4.5, 7.5, 16.5

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Email: educasia@thabyay.org
info@curriculumproject.org