MATHS MODULE 2

Algebra

Teacher's book

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1. Linear Expressions

1.1 Simplifying expressions

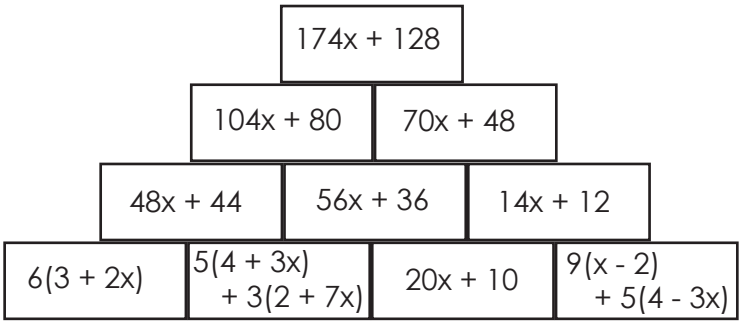
Practice - Answers

i.

a. 10x
b. 8y
c. 12x - 5
d. 8x + 3
e. 4x - 5
f. x + 3y
g. -6a - 6b
h. 4x + 8y
i. x - 4y + 4z
j. 6x + 4
k. 6y - 15
l. 14x - 18
m. 5a + 23
n. 2 + 15z

Practice - Answers

ii.



Think

Give students some time to fill the answers in the blank spaces. Then ask them some students to come and write answers on the board to check.

The answers are:

+ (+ a) = a	- (+ a) = - a
+ (- a) = - a	- (- a) = a

a x (- b) = - ab	(- a) x (- b) = ab
(- a) ÷ b = - ab	(- a) ÷ (- b) = ab

1.2 Factorising

Think

Give the students a few minutes to try and complete the sentence. Tell them to look at the examples for a hint.

The answer is:

To factorise completely the Highest Common Factor must be outside the brackets.

Practice

i.

a. 2
b. 4
c. 5
d. 12
e. 2
f. x
g. 3x

ii.

a. 5(x - 4)
b. 8(x + 3)
c. q(x + 4)
d. 8a(2x + 7)
e. 3(3x + 4y)
f. 3(4x + 3y + 1)

1.3 Algebraic fractions

Practice - Answers

a. $\frac{x}{4}$ **b.** **a** **c.** $\frac{1}{2a}$ **d.** $\frac{2}{3y}$ **e.** $\frac{3y}{7}$ **f.** $\frac{1}{x}$

g. $\frac{x}{2(x - y)}$ **h.** $\frac{x}{4(x + y)}$ **i.** $\frac{2}{3x}$ **j.** $\frac{1}{(6 - b)}$

Practice - Answers

a. $\frac{2a}{4a - b}$ **b.** $\frac{3}{5}$ **c.** $\frac{2}{3 - y}$ **d.** $\frac{6(2x - y)}{3x - y}$

Practice - Answers

a. $\frac{y + x}{xy}$ **b.** $\frac{2\mathbf{b} + \mathbf{a}}{2ab}$ **c.** $\frac{5}{6x}$ **d.** $\frac{2p - q}{8pq}$ **e.** $\frac{2\mathbf{b} - \mathbf{a}}{4ab}$ **f.** $\frac{4x + 5y}{8xy}$

g. $\frac{3(3x + 1)}{20}$ **h.** $\frac{4x + 13}{12}$ **i.** $\frac{1 - 2x}{35}$ j. $\frac{5x - 3}{42}$ k. $\frac{-1 + x}{10}$ l. $\frac{2 - 11x}{18}$

m. $\frac{17x - 1}{12}$ n. $\frac{42x - 49}{10}$

Think

Let students work in pairs to complete the statements that go with each step of the calculation.

The answers are:

$\frac{a + 2b}{8} \div \frac{2a + b}{4}$

$= \frac{a + 2b}{8} \times \frac{4}{2a + b}$ Invert the second fraction

$= \frac{4(a + 2b)}{8(2a + b)}$ Multiply the numerators and the denominators

$= \frac{a + 2b)}{2(2a + b)}$ Cancel the common factors

Practice - Answers

a. $\frac{8}{15}$ **b.** $\frac{\mathbf{ac}}{bd}$ **c.** $\frac{5(x - y)}{2x}$ **d.** $\frac{\mathbf{ac}}{b}$ **e.** $\frac{pr}{q}$ **f.** $\frac{3(\mathbf{a - b})}{4(a + b)}$

2. Linear Equations

2.1 Solving linear equations

Practice - Answers

a. The equation is: x + 4 = 9, so the number of marbles in the box is x = 5
b. The equation is: 2x + 6 = 14, so the number of marbles in the box is x = 4
c. The equation is: 8 = 2x + 2, so the number of marbles in the box is x = 3
d. The equation is: 3x + 2 = 2x + 8, so the number of marbles in the box is x = 6

Practice - Answers

i.

a. x = 4
b. a = 6
c. z = 5
d. x = 2
e. a = 6
f. x = -1

ii.

a. x = 4
b. x = 3
c. x = 6
d. x = 5
e. x = 2
f. x = 2
g. x = 2
h. x = -3
i. x = 0

iii.

a. x = 6
b. x = -6/5
c. x = 5/3
d. x = -1
e. x = 2
f. x =10

Think

Let students work in pairs to complete the statements that go with each step of the calculation

The answers are:

$3(2x + 1) + 6 + 2x = 3x - 4 - 2$	
$6x + 3 + 6 + 2x = 3x - 4 - 2$	Expand the brackets
$8x + 9 = 3x - 6$	Collect like terms
$5x + 9 = -6$	Subtract 3x from the right hand side
$5x = -15$	Subtract 9 from the left hand side
$x = -5$	Divide both sides by 5

Practice - Answers

a. $x = 5/4$
b. $x = 2$
c. $x = -1$
d. $x = 3$
e. $x = 2$
f. $x = -5$
g. $x = 11/3$
h. $p = 0$
i. $x = 5$
j. $x = 13/2$
k. $x = 3/14$

2.2 Linear equations

Practice - Answers

a. The equation is $2x + 6 = 24$. So, $x = 9$ cm
b. The equation is $x + (x + 8) = 80$. So, $x = 36$ dollars
c. The equation is $x + (x + 6) + (x + 8) = 32$. So, $x = 6$ dollars
d. The equation is $2x + 2x + 3x + 2 + 3x + 2 = 44$. So, $x = 4$ cm
e. The equation is $x + 2x + 30^\circ + 90^\circ = 360^\circ$. So, $x = 70^\circ$
f. The equation is $30 + x = 45 - 2x$. So, $x = 5$

2.3 Fractional equations

Practice - Answers

i.

a. x = 12 **b.** x = 50/6 **c.** x = 8 **d.** x = 3/20 **e.** x = 21/20 **f.** x = 55/42

ii.

a. x = 9/4 **b.** x = 55/4 **c.** x = 114/9 **d.** x = 28/17 **e.** x = 27/4 **f.** x = 20

Practice - Answers

i.

a. x = 3/4 **b.** x = 11 **c.** x = 13/7 **d.** x = 18/23 **e.** x = 12/11 **f.** x = 15/9

g. x = 7/6 **h.** x = 246/138 **i.** x = 7

ii.

a. x = -18 **b.** x = -1 **c.** x = 4/9 **d.** x = 2 **e.** x = -45/18

iii.

a. x = 8 **b.** x = -5 **c.** x = -96/10 **d.** x = 18 **e.** x = 3 **f.** x = 46/15

3. Graphing Linear Equations

3.1 Introduction

Think

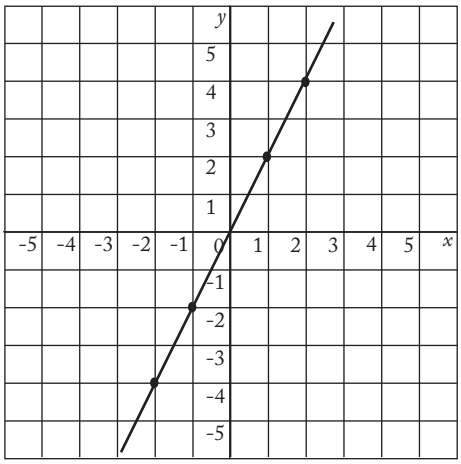
Let students work through this problem in pairs.

After they have answered the four questions, tell them to compare their answers with another groups

The answers are:

i.

x	y	(x,y)
-2	-1	(-2, -1)
2	1	(2, 1)
1	2	(1, 2)
2	4	(2, 4)



ii. See graph for the straight line

iii. The value of the y-coordinate is twice the value of the x-coordinate

iv. The answer to **iii.** tells us that the equation of the line is y = 2x

Practice - Answers

i.

a. y = 2

x	y	(x,y)
-5	-5	(-5, -5)
-2	-2	(-2, -2)
0	0	(0, 0)
1	1	(1, 1)
3	3	(3, 3)

b. y = -x

x	y	(x,y)
-5	5	(5, 5)
-2	2	(2, 2)
0	0	(0, 0)
1	1	(-1, -1)
3	3	(-3, -3)

c. y = 3x

x	y	(x,y)
-5	-15	(-5, -15)
-2	-6	(-2, -6)
0	0	(0, 0)
1	3	(1, 3)
3	9	(3, 9)

d. y = x/2

x	y	(x,y)
-5	-2.5	(-5, 2.5)
-2	-1	(-2, -1)
0	0	(0, 0)
1	0.5	(1, 0.5)
3	1.5	(3, 1.5)

e. y = x + 1

x	y	(x,y)
-5	-4	(-5, -4)
-2	-1	(-2, -1)
0	1	(0, 1)
1	2	(1, 2)
3	4	(3, 4)

f. y = 3x + 1

x	y	(x,y)
-5	-14	(-5, -14)
-2	-5	(-2, -5)
0	1	(0, 1)
1	4	(1, 4)
3	10	(3, 10)

g. y = 7x + 6

x	y	(x,y)
-5	-29	(-5, -29)
-2	-8	(-2, -8)
0	6	(0, 6)
1	13	(1, 13)
3	27	(3, 27)

h. y = 2 - x

x	y	(x,y)
-5	7	(-5, 7)
-2	4	(-2, 4)
0	2	(0, 2)
1	1	(1, 1)
3	-1	(3, -1)

i. y = 5 - 3x

x	y	(x,y)
-5	20	(-5, 20)
-2	11	(-2, 11)
0	5	(0, 5)
1	2	(1, 2)
3	-4	(3, -4)

ii. Check the students graphs to make sure they are correct

iii. Use the tables in **i.** to check that the students have the correct value for y at the point x = 0.

3.2 The gradient of a line

Practice - Answers

i.

- **a.** Gradient = 2
- **b.** Gradient = 2
- **c.** Gradient = 1
- **d.** Gradient = -2

ii. The students should notice that the gradient is always equal to the coefficient of the x term.

iii. Gradient = 4

3.4 Using graphs to solve problems

Practice - Answers

i.

a.

Time travelled in hours, x	0	1	2	3	4
Distance travelled in km, y	0	3	6	9	12

b. Check the students' graphs

c. The graphs should show that the boat travelled 7.5 km in 2 1/2 hours.

ii.

a.

Time travelled in hours, x	0	5	10	15	20	25
Distance travelled in km, y	0	8	16	24	32	40

b. Check the students' graphs

c. The graphs should show that 14 kilometres is approximately the same as 9 miles.

iii.

a.

Number of cups of rice, x	1	2	3	4	5	6
Number of cups of water, y	1.5	3	4.5	6	7.5	9

b. Check the students' graphs

c. The graph should show that Thee Thee uses 8.25 cups of water to make 5.5 cups of rice.

4. Formulae

4.1 Introduction

4.2 Constructing formulae

Think

The number of Oo's wins is (a - 3), the number of his draws is (b + 4).
The formula for Oo's total score is:

$$T_o = 3(a - 3) + (b + 4)$$

$$T_o = 3a + b - 5$$

Practice - Answers

i.

- **a.** P = 2l + 2w
- **b.** P = 3l
- **c.** P = 2l + 2m + n
- **d.** P = 3l + b + c

ii. Area = l^2 + b*1/2 = l(l + b/2)

iii. Distance = x/1000 + y + (x - a)/1000 = y + (2x - a)/1000

iv.

a. $T_m = 3x + y$

b.

Team	Win	Draw	Lose
Manchester United	x	y	z
Chelsea	x - 4	y + 6	z - 2
Liverpool	x - 8	y + 3	z + 5
Arsenal	x - 9	y + 6	z + 3

c. $T_c = 3x + y - 6$, $T_L = 3x + y - 21$, $T_A = 3x + y - 21$

4.3 Evaluating formulae

i. $T_m = 89$, $T_c = 83$, $T_L = 63$, $T_A = 63$

ii.

a. q = 7

b. b = -21

c. a = 31

d. Q = -3

iii.

a. C = 24.51
b. c = 5.00
c. 22.25

iv.

	Beijing	**Bangkok**	**Tokyo**	**New York**	**London**	**Moscow**
°C	21	35	23	-7	3	17
°F	70	95	74	19	37	63

5. Simultaneous Equations

5.1 Introduction

5.2 Solving by elimination

Practice - Answers

i.

a. x = 3, y = 2
b. a = 3, b = 5
c. x = 1, y = 7
d. x = 4, y = -3
e. x = 2, y = 5
f. p = -2, q = 1

ii.

a. x = 3, y = 1
b. p = 3, q = 4
c. x = 9, y = 1
d. p = 1, q = 0
e. x = 0, y = 6
f. x = 2, y = 3

iii.

a. x = 3, y = 2
b. x = 3, y = 0
c. p = -1, q = 4
d. x = 4, y = -2
e. x = -3, y = 2
f. x = 2, y = -4

Practice - Answers

i.

a. x = 3, y = 1
b. x = 1, y = 2
c. x = -12, y = 27
d. x = 0, y = 1
e. x = 1, y = 2
f. a = 2, b = 1
g. x = 0, y = 3
h. x = 1, y = -1

ii.

a. x = 3, y = 2
b. x = 3, y = 1
c. x = 1, y = 4
d. x = 1, y = 0
e. x = -3, y = 0
f. a = 3, b = -2
g. x = 2, y = -1
h. x = 3, y = 2

5.3 Solving by substitution

Think

Ask the students to complete the calculation by following the written steps. When they are finished ask them to compare answers with a classmate.

The answer is:

Write [2] in terms of y:	y = 7 - 2x [3]
Substitute [3] into [1]:	x - (7 - 2x) = 8
Simplify:	3x - 7 = 8
	3x = 15
Find x:	x = 5
Substitute in [1] to find y:	y = - 3
Check the solution by substituting into [2]:	2 x 5 + (-3) = 7.

This is true so the answer is correct.

Practice - Answers

a. x = 4, y = -7 **b.** m = 30, n = -5 **c.** x = -3, y = 0 **d.** x = -2, y = 5
e. s = 9, t = -4 **f.** a = 0, b = -2 **g.** x = 50, y = 50 **h.** x = 2, y = 1
i. x = 3, y = 1

5.4 Using simultaneous equations

Practice - Answers

a. The equations are x + y = 20, x - y = 4. The solution is x =12, y = 8.
b. The equations are x + y = 16, x - y = 6. The solution is x = 11, y = 5.
c. The equations are 3x + y = 33, x + 3y = 19. The solution is x =10, y = 3.
d. The equations are 2x + 3y = 2.5 tonnes, 5x + 4y = 5.2 tonnes. The solution is one elephant (x) weighs 800 kg (There are 1000 kg in a tonne).

e. The equations are x + y = 1000, 850x + 450y = 730,000. The number of adult tickets sold (x) is 700.

f. The equations are x + y = 90, x - y = 18. The solution is x =54, y = 36.

g. The equations are x + y = 20, x - y = 4. The solution is x =12, y = 8.

h. **i.** If we choose the numbers 5, 7, 9, 11, 13, 15 then the equations are 5x + 7y = 9, 11x + 13y = 15. The solution is x = -1, y = 2.

ii. If we choose the numbers 28, 21, 14, 7, 0, -7 then the equations are 28x + 21y = 14, 7x = -7. The solution is x = -1, y = 2.

The students should notice that for every set of consecutive numbers from a sequence, the solution is always the same.

5.5 Solving by drawing graphs

Practice - Answers

The students' graphs should show the following solutions:

a. x = 1.5, y = 4.5 **b.** x = 1.5, y = 5.5 **c.** x = 0.5, y = 2 **d.** x = 1.5, y = 3.5

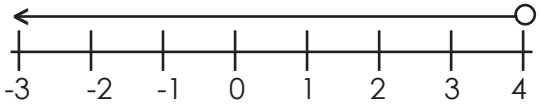
6. Linear Inequalities

6.1 Introduction

Practice - Answers

i. a. x > 1 **b.** x > -3 **c.** x ≤ -2 **d.** x ≤ 1

ii. a.



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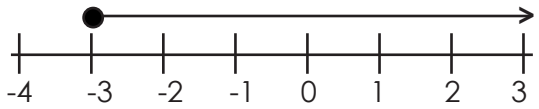

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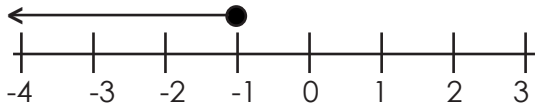

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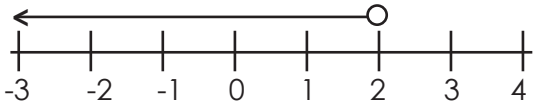

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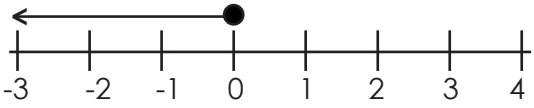

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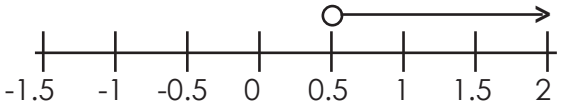


Think

Let the students work in pairs to calculate the operations **a. - f.**

a) Add 4 to both sides 3 + 4 > -6 + 4

7 > 2 True

b) Subtract two from both sides 3 - 2 > -6 - 2

1 > -8 True

c) Multiply both sides by 2 3 x 2 > -6 x 2

6 > -12 True

d) Divide both sides by 3 3 ÷ 3 > -6 ÷ 3

1 > -2 True

e) Multiply both sides by -5 3 x 5 > -6 x 5

-15 > 30 False

f) Divide both sides by - 3 3 ÷ -3 > -6 ÷ -3

-1 > 2 False

When they have completed the operations, they should be able to complete the statements.

The answers are:

To solve an inequality you **can**:

Add the same number to both sides

Subtract the same number from both sides

Multiply both sides by the same positive number

Divide both sides by the same positive number

To solve an inequality we **cannot**:

Multiply both sides by the same negative number

Divide both sides by the same negative number

6.2 Solving inequalities

Practice - Answers

i.

a. $x < 12$ **b.** $x < 2$ **c.** $x < 3$ **d.** $x < -7$ **e.** $x < -2$ **f.** $x < -3$ **g.** $x > 12$

ii.

a. $x < 3$ **b.** $x > 2$ **c.** $x < 9/4$ **d.** $x > 3/2$ **e.** $x < 1$ **f.** $x < 1$ **g.** $x > -2$

h. $x < -1$ **i.** $x > 1$ **j.** $x > 0$ **k.** $x < -3$

Practice - Answers

a. $x > 11$ **b.** $x > 4$ **c.** $x \leq -7/2$ **d.** $x \geq 5/2$ **e.** $x < -1$ **f.** $x < -4/3$ **g.** $x \geq 1/2$

6.3 Range of values

Practice - Answers

a. $x > 3$ **b.** $2 < x < 3$ **c.** No values of x **d.** $-2 < x < 4$ **e.** $x < 0$
f. No values of x **g.** $-3 < x < -1$ **h.** No values of x

Practice - Answers

a. $x < 12, x > -1, -1 < x < 12$ **b.** $x \leq -1, x \geq -3$, No values of x **c.** $x \leq 7, x \geq -2, -2 \leq x \leq 7$
d. $x > 1, x < 2, 1 < x < 2$ **e.** $x > 2, x < 3, 2 < x < 3$ **f.** $x < 2, x > -1, -1 < x < 2$

Practice - Answers

a. $2 < x < 5$ **b.** $x < -2$ **c.** $x > 1$ **d.** $-4 < x < 2$ **e.** $9/5 < x < 3$
f. $1/2 < x < 1$

6.4 Inequalities and regions

Practice - Answers

Check the students' diagrams to make sure they have the correct answers

Practice - Answers

i. Check the students' diagrams to make sure they have the correct answers

ii. **a.** No **b.** No **c.** No **d.** Yes **e.** No **f.** No

iii. **a.** $x < 2$ **b.** $x < -1$ **c.** $y < 3$ **d.** $3 < x < 2$

6.5 Inequalities with 2 variables

Practice - Answers

Check the students' diagrams to make sure they have the correct answers

Practice - Answers

a. $y - x < 2$ **b.** $x + y \leq 3$ **c.** $x + 2y \leq 2$ **d.** $x + y < 2$

7. Quadratic Expressions

7.1 Introduction

Practice - Answers

a. $7a^2 - 9b^2 + 2c$
b. $7a^2 - 9b^2 + 2c$
c. $-2(a^2 - 5b^2 - 3c^2 + 2d^2)$
d. $5z^2 - 5x^2$
e. $14a^2 + 2ab$
f. $2mn$
g. $2x^2 - x$
h. $2x^2 - 9x + 10$
i. $20y^2 - 23y + 6$
j. $3x^2 + 7xy - 4y^2$

Think

Students should remember from Module 1 that this law is called the commutative law of multiplication. It tells us we can multiply terms in any order.

This is the commutative law of multiplication.

7.2 Brackets

Practice - Answers

i. a. $x^2 + 3x + 2$ **b.** $x^2 + 10x + 16$ **c.** $x^2 + 6x + 9$ **d.** $x^2 + ax + bx + ab$

ii. a. $ab + 3a + 4b + 12$ **b.** $x^2 + 8x + 12$ **c.** $b^2 + 11b + 28$ **d.** $2ab + 8a + 3b + 12$

e. $3b^2 + 10b + 8$ **f.** $3b^2 - 2b - 8$ **g.** $x^2 + 4x - 12$ **h.** $ab - 3a - 4b + 12$

i. $x^2 - 11x + 28$ **j.** $b^2 + 8b + 12$ **k.** $ab + 3a - 4b - 12$ **l.** $b^2 - 3b - 28$

iii. a. $3ab - 4b + 15a - 20$ **b.** $2x^2 + 5xy + 3y^2$ **c.** $6b^2 + 23bc + 20c^2$

d. $a^2 + 6a + 9$ **e.** $2a + b + 2$ **f.** $9x^2 - 1$ **g.** $49x^2 - 4$

h. $5x^2 - 20x - 60$ **i.** $4a^2$

Think

i. Let the students work in pairs to find the identities using the FOIL rule. Check that they all have the correct answer before going to question **ii.**

The answers are: $(x + a)^2 = (x + a)(x + a) = x^2 + ax + ax + a^2 = x^2 + 2a + a^2$

$(x - a)^2 = (x - a)(x - a) = x^2 - ax - ax + a^2 = x^2 - 2a + a^2$

$(x + a)(x - a) = x^2 - ax + ax + a^2 = x^2 + a^2$

ii. When everyone has the correct answers to **i.**, let them work in the same pairs to complete the statements:

a. $(x + a)^2$

Square of the first term + twice the product of the two terms + Square of the second term

b. $(x - a)^2$

Square of the first term - twice the product of the two terms + Square of the second term

Practice - Answers

i. a. $x^2 + 2x + 1$ **b.** $x^2 + 4x + 4$ **c.** $b^2 + 8b + 16$ **d.** $t^2 + 20t + 100$

e. $x^2 + 2xy + y^2$ **f.** $p^2 + 2pq + q^2$ **g.** $u^2 + 2uv + v^2$

ii. a. $x^2 - 4x + 4$ **b.** $x^2 - 12x + 36$ **c.** $x^2 - 2xy + y^2$ **d.** $x^2 - 14x + 49$

e. $a^2 - 2ab + b^2$ **f.** $9x^2 - 6x + 1$ **g.** $16y^2 - 8y + 1$

iii. a. $x^2 - 16$ **b.** $x^2 - 144$ **c.** $x^2 - 64$ **d.** $4x^2 - 1$

e. $25x^2 - 16$

Practice - Answers

a. $(x + 4)^2 = x^2 + 8x + 16$ **b.** $(x - 10)^2 = x^2 - 20x + 100$

c. $(x + 1)^2 = x^2 + 2x + 1$ **d.** $(x - 12)^2 = x^2 - 24x + 144$

7.3 Factorising quadratic expressions

Practice - Answers

a. $4x(2x + 1)$ **b.** $3p(2p + 1)$ **c.** $5c(3 - 2c)$ **d.** $ax(x + 1)$

e. $6ab(2a + 3b)$ **f.** $2xy(2x - y)$ **g.** $4ab(a + 2b + 3)$ **h.** $2xy(2x + 3y - 1)$

i. $3ax(4x + 2a - 1)$ **j.** $abc(a + b + c)$

Practice - Answers

i. a. (x + 1)(x + 5) **b.** (x + 3)(x + 4) **c.** (x + 4)(x + 13)
d. (x - 1)(x - 8) **e.** (x - 7)(x - 4) **f.** (x - 2)(x - 16)

ii. Make sure the students return to the original expression when they expand the brackets.

iii. a. (x + 3)(x + 4) **b.** (x + 1)(x + 20) **c.** (x + 1)(x + 7) **d.** (x + 1)(x + 12)
e. (x + 3)(x + 12) **f.** (x + 2)(x + 20) **g.** (x + 4)(x + 10) **h.** (x - 1)(x - 8)
i. (x - 3)(x - 4) **j.** (x - 4)(x - 7) **k.** (x - 2)(x - 16) **l.** (x - 7)(x - 9)

Think

i. Give the students a short time to write the correct signs

The answer is: $x^2 + 5x - 24 = (x - 8)(x - 3)$

ii. Complete the statement

If the x term is positive and the number term is positive then the signs in the brackets are positive.

If the x term is positive and the number term is negative then the signs in the brackets are negative.

Practice - Answers

a. (x + 7)(x - 2) **b.** (x + 6)(x - 5) **c.** (x + 3)(x - 9) **d.** (x + 20)(x - 4)
e. (x - 4)(x + 12) **f.** (x + 3)(x - 14) **g.** (x + 12)(x - 5) **h.** (x - 2)(x + 14)
i. (x - 7)(x + 6) **j.** (x + 9)(x - 7)

Think

Here the students have to put the different combinations of factors in the brackets until they reach the correct factorisation.

The answer is:

(15x + 8)(x + 1) middle term is 23x which is incorrect.
(15x + 4)(x + 2) middle term is 34x which is incorrect.
(15x + 2)(x + 4) middle term is 62x which is incorrect.
(15x + 1)(x + 8) middle term is 121x which is incorrect.
(5x + 1)(3x + 8) middle term is 43x which is incorrect.
(5x + 2)(3x + 4) middle term is 26x which is correct.

The final answer is $15x^2 + 26x + 8 = (5x + 2)(3x + 4)$

Practice - Answers

i. Factorise

a. 2(x + 3)(x + 4) **b.** 7(x + 1)2 **c.** 4(x + 3)(x - 4) **d.** 3(x + 2)(x + 6) **e.** 5(x + 2)(x - 3)

ii. Factorise

a. (3x +2)(2x + 1) **b.** (3x +1)(5x + 2) **c.** (7x +2)(5x + 2) **d.** (5x - 2)(3x + 1)

e.(3x + 4)(8x - 5) **f.** (3x - 1)(3x - 4) **g.** (2x - 1)(8x - 1) **h.** (5x - 3)(3x - 7)

i. (3a - 5)(2a + 3) **j.** $(3b - 2)^2$

Practice - Answers

a. $(x + 5)(x - 5)$ **b.** $(x + 2)(x - 2)$ **c.** $(x + 8)(x - 8)$ **d.** $(x + 7)(x - 7)$
e. $(3 + x)(3 - x)$ **f.** $(6 + x)(6 - x)$ **g.** $(5 + x)(5 - x)$ **h.** $(y + x)(y - x)$

7.4 Algebraic fractions

Think

Let the students follow the written steps to simplify the expression.

$$\frac{2x - 4}{x^2 - 5x + 6} \div \frac{3x + 9}{x^2 - 9}$$

The answer is:

Factorise $\quad \frac{2(x - 2)}{(x - 2)(x - 3)} \div \frac{(x + 3)}{(x + 3)(x - 3)}$

Cancel the common factors $\quad \frac{2}{(x - 3)} \div \frac{3}{(x - 3)}$

Invert the second fraction $\quad \frac{2}{(x - 3)} \times \frac{(x - 3)}{3}$

Cancel the common factors $\quad \frac{2}{3}$

The final answer is $\quad \frac{2x - 4}{x^2 - 5x + 6} \div \frac{3x + 9}{x^2 - 9} = \frac{2}{3}$

Practice - Answers

a. $\frac{x + 2}{2}$ **b.** $\frac{x - 1}{x + 4}$ **c.** $\frac{5}{(x + 1)}$ **d.** $\frac{3(x - 17)}{(x + 5)(x - 5)}$ **e.** $\frac{9x + 58}{(x + 7)(x - 7)}$

f. $\frac{1}{(x + 1)}$ **g.** $\frac{1}{(x + 1)(x + 2)}$ **h.** $\frac{x(x + 2)}{(x - 2)^2}$ **i.** 2

8. Pythagoras' Theorem

8.1 Introduction

Practice - Answers

The students can leave their answers in surd form if there are no calculators

a. √106 m = 10.3 m **b.** √233 m = 15.3 m **c.** √10.4 m = 3.22 m

d. √18628 m = 136 cm **e.** √530 m = 23 cm **f.** √10405 m = 102 cm

Practice - Answers

a. $\sqrt{81}$ m = 9 m
b. $\sqrt{576}$ m = 24 m
c. $\sqrt{56.25}$ m = 7.5
d. $\sqrt{110.25}$ m = 10.5 cm

8.2 Using Pythagoras' theorem

Practice - Answers

The students can leave their answers in surd form if there are no calculators

a. √80 m = 8.94 units
b. √450 m = 21.2 cm
c. i. √5.76 cm = 2.4, ii. √2.08 cm = 1.44 cm
d. x = √676 cm = 26 cm, y = √225 cm = 15 cm
e. √185 m = 13.6 units
f. √39 m = 6.24 m
g. x = 5 cm

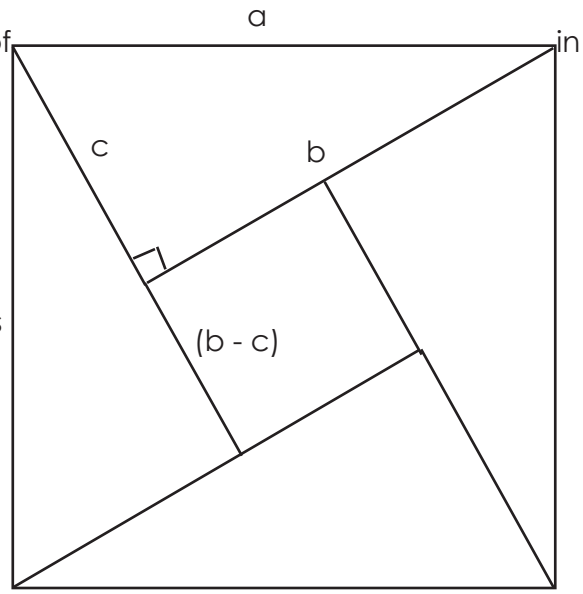
8.3 Proof of Pythagoras' theorem

Think

Let students work through this proof in pairs. They should be able to use the information given to derive this proof:

$$a^2 = (b - c)^2 + 4 \times \frac{1}{2} bc$$
$$= b^2 - 2bc + c^2 + 2bc$$
$$= b^2 + c^2$$

When they have finished ask one pair of students to come to the board and write out their proof. Ask the class if there are any mistakes.



9. Quadratic Equations

9.1 Introduction

Think

This exercise shows the students that there are two solutions to a quadratic equation. Give them a short time to fill the gaps and then check their answers to make sure they understand.

The answer is:

If we let X = (x - a) and Y = (x - b) then we have the quadratic equation: (x - a)(x - b) = 0.

This quadratic equation is only true if: (x - a) = 0 or/and (x - b) = 0.

Practice - Answers

i.

a. x = 0, x = 3 **b.** x = 0, x = 5 **c.** x = 0, x = -4 **d.** x = 0, x = -5

e. x = 0, x = 7 **f.** x = 0, x = -9

ii.

a. x = 1, x = 2 **b.** x = 10, x = 7 **c.** x = 6, x = 1 **d.** x = -1, x = -8

e. x = a, x = b

iii.

a. x = 5/2, x = 1 **b.** x = 4/5, x = 3/4 **c.** x = 0, x = 3/10 **d.** x = -5/6, x = 2/8

e. x = -3/4, x = -3/2

9.2 Solving by factorising

i.

a. x = 1, x = 2 **b.** x = 2, x = 3 **c.** x = 3, x = 4 **d.** x = 1, x = -7

e. x = 3, x = -4 **f.** x = -3, x = 8 **g.** x = -1, x = -2 **h.** x = -3, x = -6

i. x = -1, x = -13 **j.** x = -1, x = -15

ii.

a. x = 1, x = -1 **b.** x = 4, x = -4 **c.** x = 2, x = -2 **d.** x = 7, x = -7

e. x = 9, x = -9 **f.** x = 12, x = -12

iii.

a. x = 1/2, x = 2 **b.** x = -1, x = -2/3 **c.** x = 2/3, x = 3 **d.** x = -2/5, x = -1 1/3

e. x = -1/3, x = 2 1/2 **f.** x = 3/4, x = 1 1/2 **g.** x = -1/2, x = -1 1/2 **h.** x = 3 1/2, x = -3/5

iv.

a. x = 5, x = -3 **b.** x = 3, x = -4 **c.** x = 1/2, x = -1/3 **d.** x = 1, x = 4

e. x = 5, x = 7

Practice - Answers

a. x = -2, x = -3 **b.** x = -4, x = 3 **c.** x = -5/2, x = 5 **d.** x = 1, x = 2

e. x = 4 1/3, x =2

Practice - Answers

a. The equation is $x(x - 4) = 140$, so $x = 10$ or -14. Ann is 10 years old (the age cannot be negative)

b. The equation is $x(x + 4) = 77$, so $x = 7$ or -11. The rectangle is 7 cm wide (the width cannot be negative) and 11 cm long.

c. The equation is $x(x + 5) = 66$, so $x = 6$ or -11. The rectangle is 6 cm wide (the width cannot be negative) and 11 cm long.

d. The equation is $x(x + 4) = 45$, so $x = 5$ or -9. The rectangle is 5 cm wide (the width cannot be negative) and 9 cm long.

e. Using Pythagoras' theorem we have $(x + 8)^2 = x^2 + (x + 7)^2$. Solving gives $x = -3$ or $x = 5$. The length of the sides must be positive so $x = 5$ cm. The length of the sides are 5 cm, 12 cm and 13 cm.

f. The equation is $(3x + 1)(x + 2) - x^2 = 62$. Solving gives $x = -15/2$ or $x = 4$. $x = 4$ gives positive values for the length of the sides. The width is 6 cm and the length is 13 cm.

9.3 Completing the square

Practice - Answers

i.

a. $(x + 2)^2 - 4$ **b.** $(x - 7)^2 - 49$ **c.** $(x + 1/2)^2 - 1/4$ **d.** $(x - 2)^2 - 4$

e. $(x - 5)^2 - 25$

ii.

a. $2(x + 4)^2 - 32$ **b.** $3(x - 2)^2 - 12$ **c.** $5(x - 3/2)^2 - 45/4$ **d.** $2(x + 1/4)^2 - 1/2$

e. $7(x - 2)^2 - 28$

9.4 Solving by completing the square

Think

Let the students work in pairs to follow the steps to complete the square for $x^2 + 10x + 18 = 0$.
The answer is:

Subtract 18 from both sides:	$x^2 + 10x = -18$
Complete the square for $x^2 + 10x$ (see previous page):	$(x + 5)^2 - 5^2 = -18$
Add 25 to both sides:	$(x + 5)^2 = 7$
Square root both sides:	$x + 5 = \pm \sqrt{7}$
Subtract 5 from both sides:	$x = -5 \pm \sqrt{7}$

Practice - Answers

a. $5 \pm \sqrt{22}$ **b.** $4 \pm \sqrt{18}$ **c.** $\frac{-9 \pm \sqrt{69}}{2}$ **d.** $1 \pm \sqrt{\frac{2}{3}}$

f. $1 \pm \sqrt{\frac{9}{2}}$ **g.** $\frac{-5 \pm \sqrt{61}}{6}$ **h.** $\frac{7 \pm \sqrt{65}}{4}$ **i.** $\frac{4 \pm \sqrt{19}}{3}$

9.5 The quadratic formula

Practice - Answers

i. a. x = -1, x = -5 **b.** x = -1, x = -4 **c.** x = -1, x = -8 **d.** x = -11, x = 1

e. x = -10, x = -1 **f.** x = 2, x = 5 **g.** x = -2, x = 9 **h.** x = -2, x = 6

i. x = -6, x = -2

ii. a. $\frac{-7 \pm \sqrt{33}}{4}$ b. $\frac{-7 \pm \sqrt{33}}{8}$ c. $\frac{-9 \pm \sqrt{41}}{10}$ d. $\frac{-7 \pm \sqrt{17}}{4}$ e. $\frac{-7 \pm \sqrt{69}}{6}$

iii. a. $\frac{-7 \pm \sqrt{61}}{6}$ b. $\frac{9 \pm \sqrt{57}}{6}$ c. $\frac{1 \pm \sqrt{33}}{16}$ d. $\frac{-2 \pm \sqrt{7}}{3}$

iv.

a. The equation is $x^2 + 3x - 20 = 0$. The solutions are $x = -6.2$, $x = 3.2$. So the sides are 3.2 cm and 6.2 cm long.

b. The equation is $x^2 - 4x - 8$. The solutions are $x = -2.9$, $x = 10.9$. So the hypotenuse is 13,9 cm long.

c. The equation is $x^2 + 2x - 31$. The solutions are $x = -6.7$, $x = 4.7$. So x is 4.7.

9.6 Graphing quadratic equations

Think

Let students work through this problem in pairs.

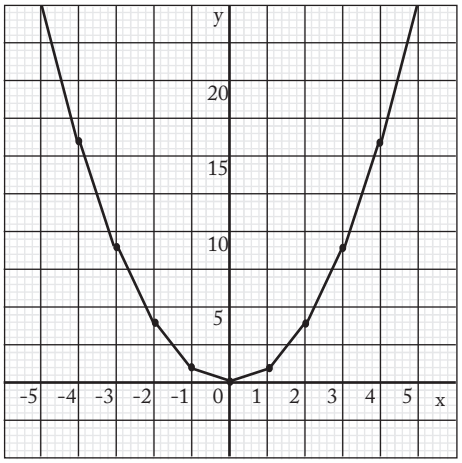
After they have answered three questions, tell them to compare their answers with another group.

The answers are:

a.

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

b. and **c.**



Practice - Answers

Check the students' graph to make sure they are correct.

9.7 Solving quadratic equations by graphing

Practice - Answers

The students' graphs should show the following solutions

a. x = 5.6, x = 1.4 **b.** x = 2.3, x = 1.3 **c.** x = 2.8, x = -1.3 **d.** x = 1.3, x = 0.3

10. Sequences

10.1 Introduction

Practice - Answers

i.

- **a.** 25, 36, … Square the next natural number
- **b.** 15, 18, … Add 3 to the previous term, or multiply the next natural number by 3
- **c.** 35, 42, … Add 7 to the previous term, or multiply the next natural number by 7
- **d.** -1, -5, … Take 4 from previous term
- **e.** 0.125, 0.0625, … Divide previous term by 2
- **f.** 30, 42, … Add to the previous term a number that increases 2 each time

ii.

- **a.** 15, …, 33, …
- **b.** 12, …

iii. 15, 21, 28, 36, …

iv. 1, 4, 9, 16, 25, …

10.2 The nth term of a sequence

Practice - Answers

i.

a. 3, 5, 7, 9, … 15 **b.** 1, 3, 5, 7, … 13 **c.** 2, 4, 8, 16, … 128

d. 0, 3, 8, 15, … 48 **e.** 5, 6, 7, 8, … 11 **f.** 5, 7, 9, 11, … 17

ii.

a. 10, 13, 16, 19, 22, … **b.** 13, 11, 8, 7, 5 … **c.** 6, 9, 14, 21, 30, …

d. 1/2, 1/4, 1/8, 1/16, 1/32, … **e.** 1, 1/2, 1/3, 1/4, 1/5, …

f. 1/2, 2/3, 3/4, 4/5, 5/6, …

10.3 Finding the formula for a sequence

Practice - Answers

a. 3n **b.** -n **c.** n + 1 **d.** 4n **e.** 2n + 5 **f.** 3n - 3 or 3(n - 1)
g. 1/(n + 2) **h.** n(n + 2) **i.** n^3 **j.** 4 - n

11. Indices

11.1 Introduction

Practice - Answers

i.

a. 64 **b.** 100,000 **c.** 625 **d.** 343 **e.** 20,736

ii.

a. 10^3 **b.** 5^3 **c.** 8^3 **d.** 5^4

iii. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$. Since $2 \times 2 = 4$, then $64 = 4 \times 4 \times 4 = 4^3$

11.2 The index laws

Practice - Answers

i.

a. $5^6 = 15{,}625$ **b.** $5^8 = 390{,}625$ **c.** $10^2 = 100$ **d.** $12^3 = 1728$
e. 1 **f.** $2^{10} = 1024$ **g.** 6 **h.** $3^{-4} = 1/81$
i. $5^5 = 3125$ **j.** $2^2 = 4$ **k.** 3^{-4} **l.** 2^{-4} **m.** $3^2 = 9$

ii. a. a^2 **b.** $x^{3/2}y^{5/2}$ **c.** $3x$ **d.** $2/x$ **e.** $a^2/5$
f. $729x^3$ **g.** 16 **h.** $24y^2$

Practice - Answers

a. 2 **b.** 1/2 **c.** 8 **d.** 2 **e.** 1/8 **f.** 1/5 **g.** 10
h. 125 **i.** 8

11.3 Equations involving indices

Practice - Answers

a. 1/2 **b.** 0 **c.** -3 **d.** 2/3 **e.** 5 **f.** 4 **g.** 3
h. 1/2

Glossary of Keywords

The glossary in the Students' book is a list of all mathematical words that appear in the module. They are given in the order that they appear.

The following short activities are added to this guide to help students remember mathematical vocabulary. They can be used in several ways: to test prior knowledge of a topic, as warm-up activities at the beginning of a lesson or to review what has been learnt at the end of a topic.

Activity 1 - Discuss questions in pairs

Students are given questions to discuss that relate to a topic.
Example questions –
What does infinite mean?
How do I draw a graph of a linear equation?
What are the different methods we use to solve quadratic equations?
What is the commutative law?
What is a coefficient?

Activity 2 - True or false

Students work in pairs to decide if statements about a topic are true or false.

Example for fractions –
The graph of a quadratic equation is called a hyperbola.
A quadratic equation has a degree of 2.

Activity 3 - Give an explanation

Students work in pairs to prepare a short explanation to questions. Ask some students to give their explanation to the class.

Examples –
Explain how to solve simultaneous equations by substitution.
Explain how we use Pythagoras' theorem to find the length of the hypotenuse
Explain the mistake in this statement:

Activity 4 - Brainstorming

Write a topic on the board and ask students what they know about the topic. Write their answers on the board.

Activity 5 - What's the topic?

Write words linked to a topic on the board and ask students if they can guess the topic.

Assessment

This is assessment covers most of the topics in this module and should give you an idea of how much the students have understood. It is recommended that you give it as a class test, with some time for review and revision beforehand.

Part 1 - Answers

Each question in part 1 is worth 1 mark.

a. hypotenuse **b.** degree **c.** coordinates **d.** gradient **e.** parabola
f. factorise **g.** linear **h.** coefficient **i.** general rule **j.** sequence

Total for part 1: 10 marks

Part 2 - Answers

The total mark for the question is given on the right hand side of the page.

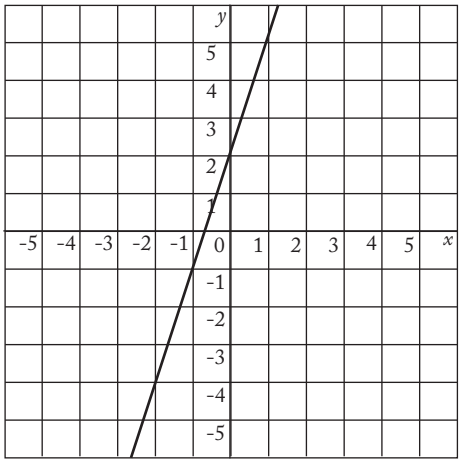
1. a) 5m b) 14a + 2b c) 4p - 4q **3 marks**

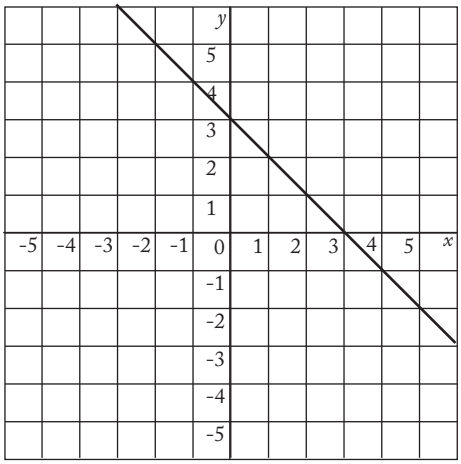
2. a) 2(2p - 3) b) 2(3a + 2b) c) 2a(b + c) **3 marks**

3. a) x = -3 b) t = -3 c) x = 2 **3 marks**

4. a) p = 60 b) n = 15 c) p = 6 5/8 **3 marks**

5. a) b) **4 marks**





6. a) p = 30.2 b) x = 0.8 c) u = 4.2 **3 marks**

7. a) x = 4, y = 1 b) x = 7, y = -1.5 c) x = 4, y = 1 **6 marks**

8. a) x < -3 b) j > -1 c) x > 2 **3 marks**

9. a) $6x^2 + 23xy + 20y^2$ b) $12p^2 + 14p + 4$ c) $4x^2 + 4x + 1$ **3 marks**

10. a) (x - 4)(x - 1) b) (x + 3)(x + 2) c) 2(3a + 2)(a + 1) **3 marks**

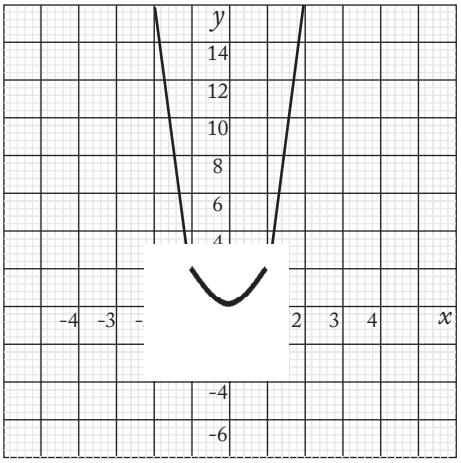
11. a) $\frac{x - 1}{x + 4}$ b) $\frac{4}{(x + 1)(x + 2)}$ c) 2 **4 marks**

12. a) 13 cm b) 12.5 cm **4 marks**

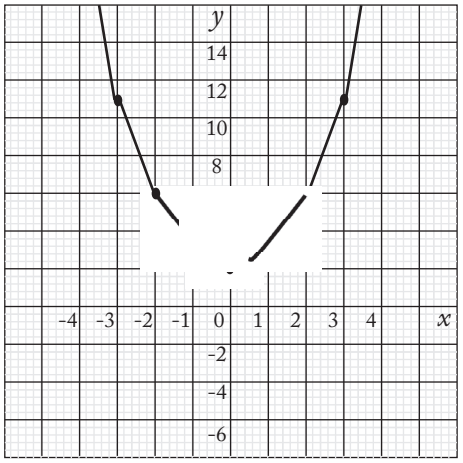
13. a) y = 4, y = -1 b) y = -3, y = 1/2 c) y = -1/2, y = 7/3 **6 marks**

14. a) $2 \pm \sqrt{3}$ b) $\frac{5 \pm \sqrt{21}}{2}$ c) $\frac{9 \pm \sqrt{65}}{8}$ **6 marks**

15. a)



b) **4 marks**



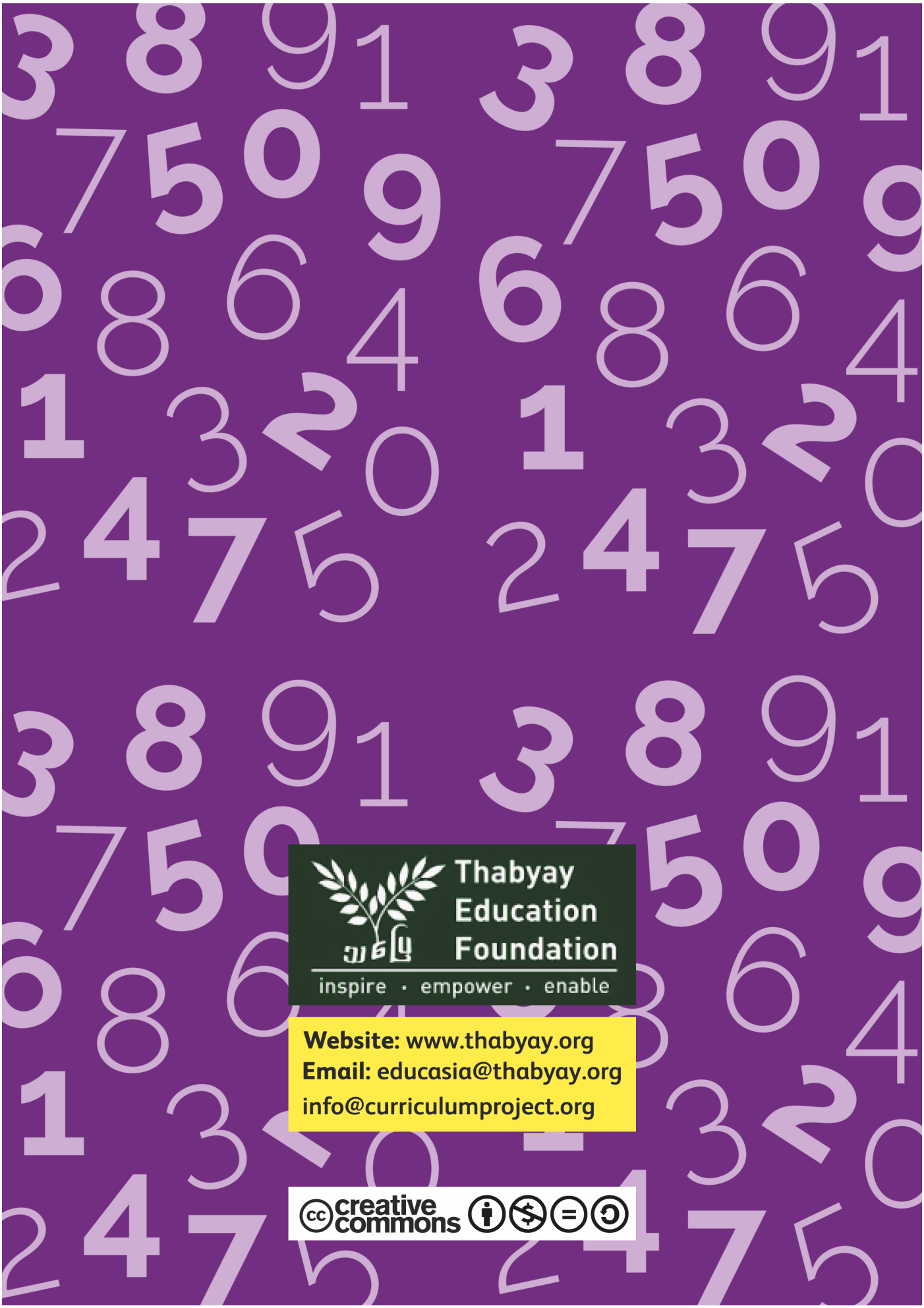
16. a) 3, 4, 5, 6, 7 b) 2, 11, 26, 47, 74 c) 4, 9, 16, 25, 36 **3 marks**

17. a) 4n + 3 b) 14 - 2n c) 10 - 3n **6 marks**

18. a) $18a^2b^3c^5$ b) $63x^8$ c) $6xy^3$ **3 marks**

Total for part 2: 70 marks

Total for test: 80 marks



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Website: www.thabyay.org
Email: educasia@thabyay.org
info@curriculumproject.org